Parallel Algorithm Design

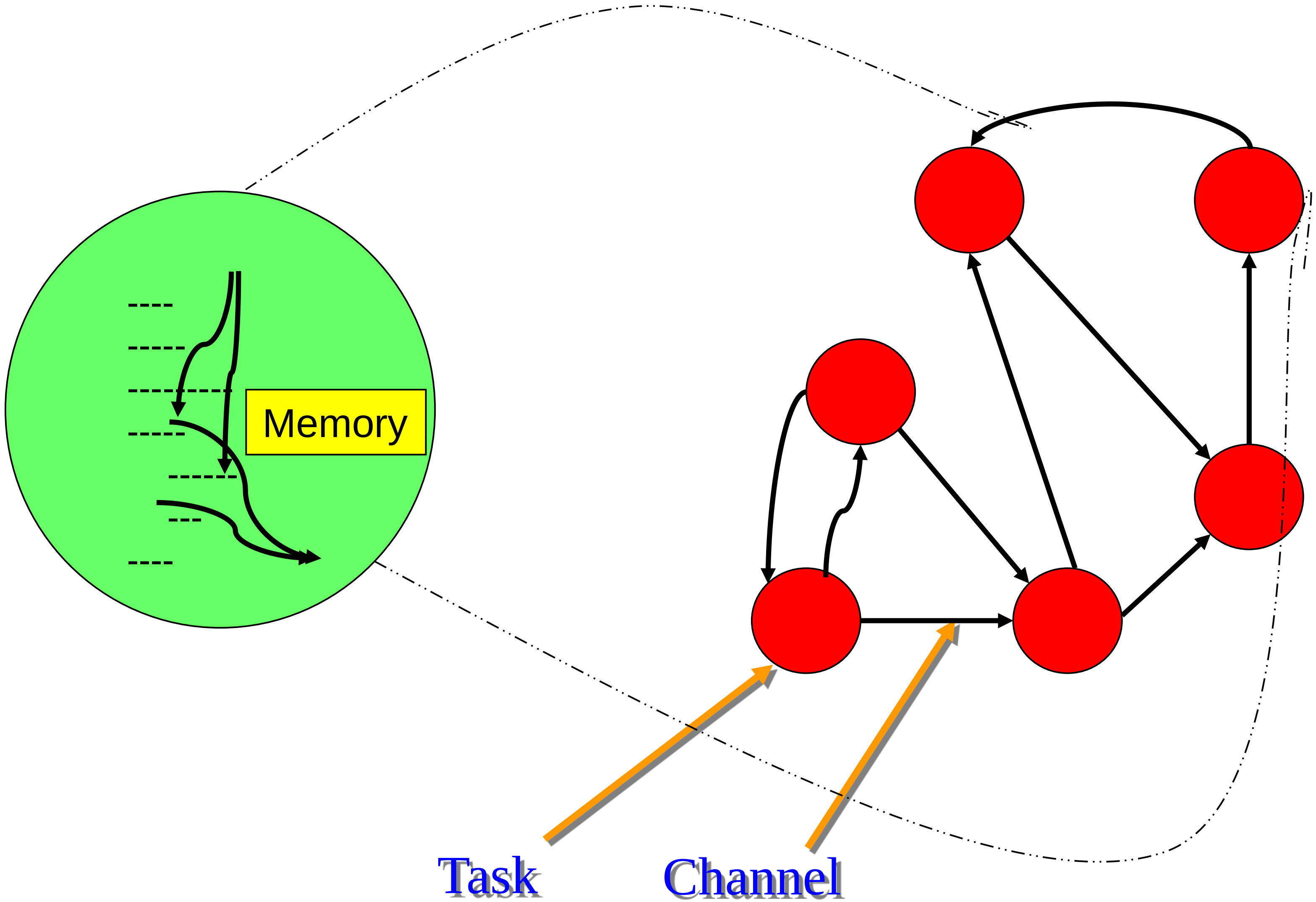
Outline

- Task/channel model
- Algorithm design methodology
- Case studies

Task/Channel Model

- Parallel computation = set of tasks
- Task
  - Program
  - Local memory
  - Collection of I/O ports
- Tasks interact by sending messages through channels
- At input port task must wait until the value appears, means task is **blocked**
- In this model receiving is a **synchronous**, while sending is an **asynchronous** operation

Task/Channel Model



Source: M.J. Quinn

Foster's Design Methodology*

- Partitioning
- Communication (Concentration on inherent parallelism)
- Agglomeration
- Mapping (Concentration on implementation on real HW)

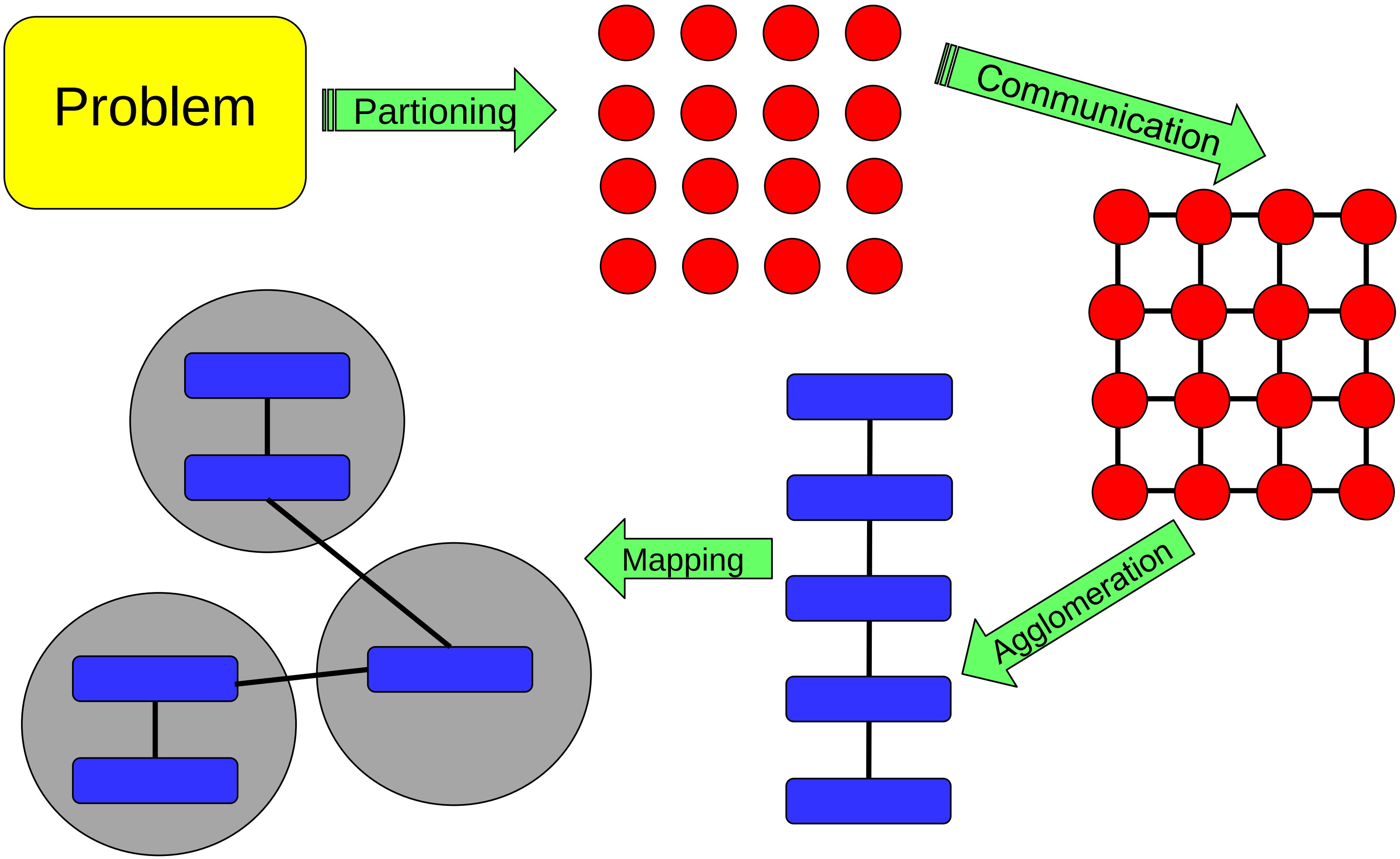
"It delays the machine dependent considerations at the later stage "

* Foster, Ian. *Designing and Building Parallel Programs: Concepts and Tools for Parallel Software Engineering*, Reading, MA: Addison-Wesley, 1995

Foster’s Design Methodology

A well-known design methodology that is used to architect and implement distributed-memory systems (DMS) particularly on Multi computer systems.

Foster's Methodology



Source: M.J. Quinn

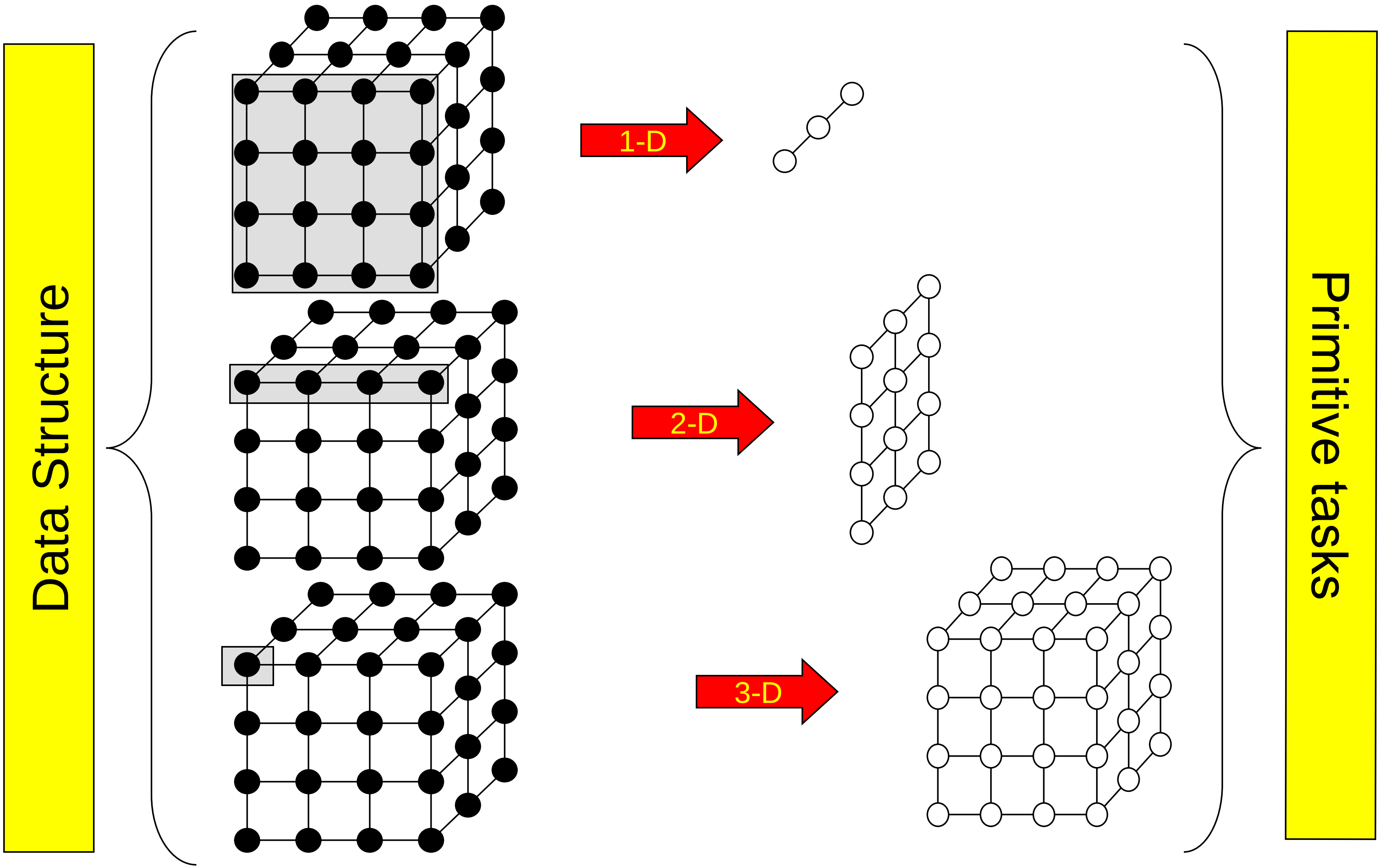
1.Partitioning

- Dividing computation and data into pieces
- Domain decomposition
  - Divide data into pieces
  - Determine how to associate computations with the data
- Functional decomposition
  - Divide computation into pieces
  - Determine how to associate data with the computations

1.Partitioning

- Focus is on most frequently accessed data structure
- In the matrix, we can partition data into
  - Collection of 2-D slice: resulting in a 1-D collection of primitive tasks
  - Collection of 1-D slice: resulting in a 2-D collection of primitive tasks
  - Consider each elements of the matrix individually: 3-D collection of primitive tasks

Domain Decompositions



Source: M.J. Quinn

1.Partitioning Checklist

- At least 10x more primitive tasks than processors in target computer
- Minimize redundant computations and redundant data storage (If not, design does not work well when problem size increases)
- Primitive tasks roughly the same size (If not, load balancing problem)
- Number of tasks an increasing function of problem size (If not, not scale well)

2.Communication

- Determine values passed among tasks
- Local communication
  - A task needs values from a small number of other tasks
  - Create channels illustrating data flow
- Global communication
  - Significant number of tasks contribute data to perform a computation
  - Don't create channels for them early in design

2.Communication Checklist

Communications are overhead in parallel algorithms, minimizing them is an important goal

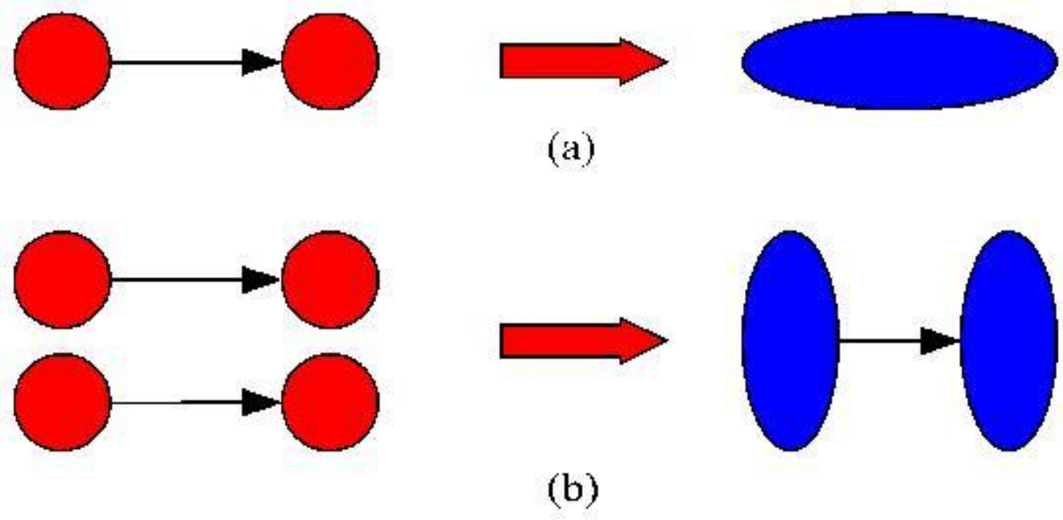
- Communication operations balanced among tasks
- Each task communicates with only small group of neighbors
- Tasks can perform communications concurrently
- Task can perform computations concurrently

3.Agglomeration

- Grouping tasks into larger tasks
- Goals
  - Improve performance
  - Maintain scalability of program
  - Simplify programming
- *In MPI programming, goal often is to create one agglomerated task per processor*

Agglomeration Can Improve Performance

- Eliminate communication between primitive tasks agglomerated into consolidated task (specially when the tasks cannot perform their operations in parallel)
- Combine groups of sending and receiving tasks (for reducing message latency)

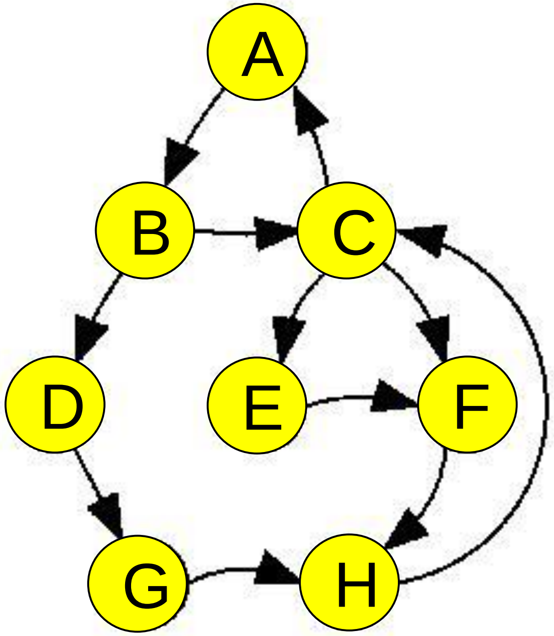


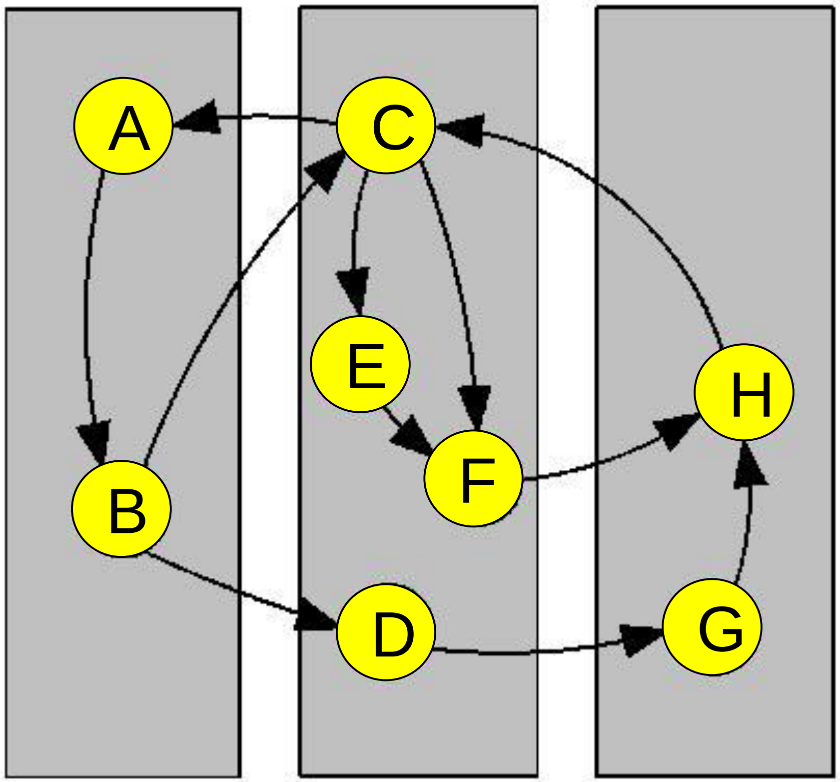
Source: M.J. Quinn

4.Mapping

- Process of assigning tasks to processors
- *Centralized multiprocessor: mapping done by operating system*
- *Distributed memory system: mapping done by user*
- Goals of mapping
  - Maximize processor utilization
  - Minimize inter-processor communication

Mapping Example





Source: M.J. Quinn

Optimal Mapping

- Finding optimal mapping is NP-hard
- Must rely on heuristics

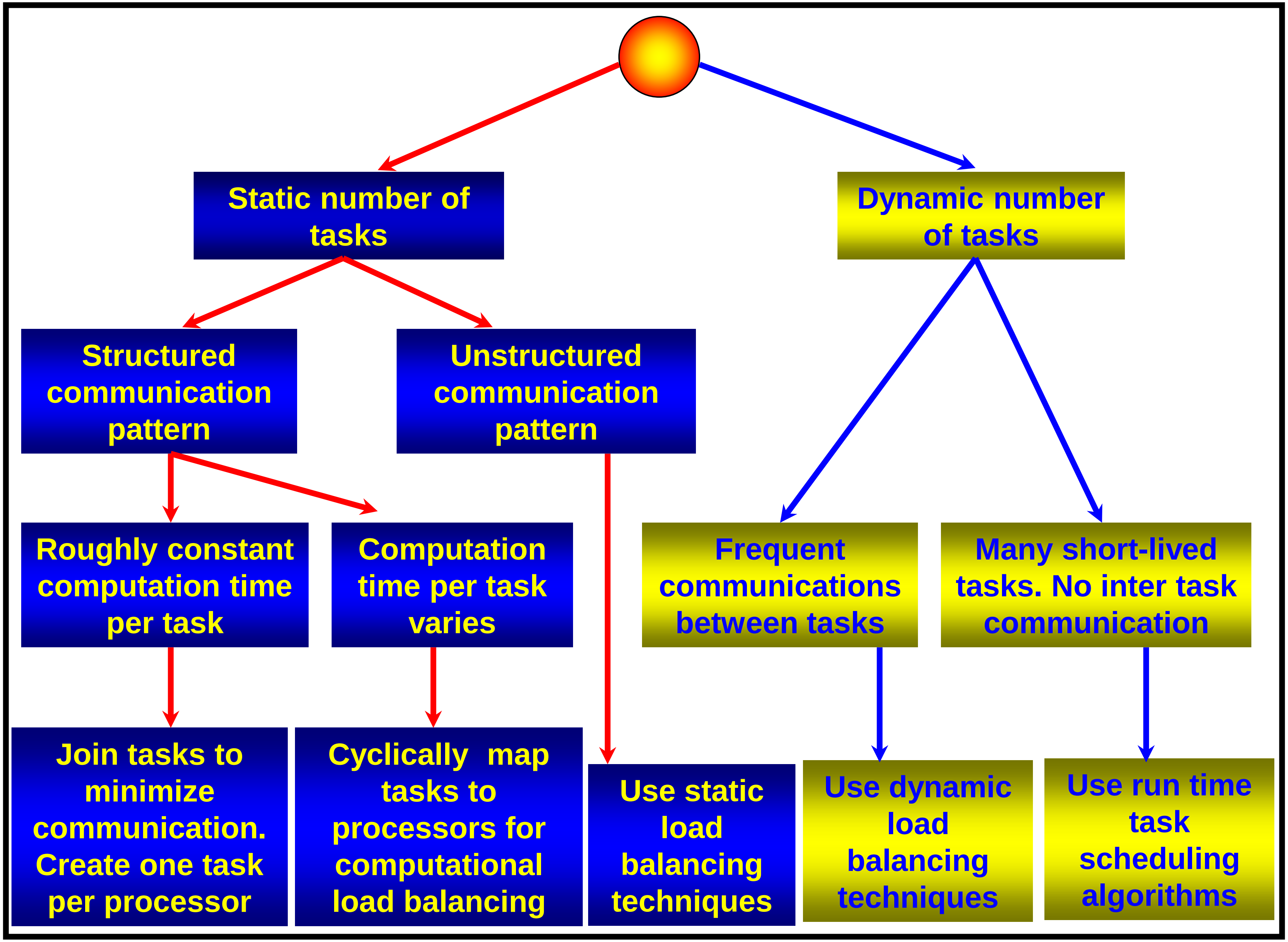
Definition

- In *local* communication, each task communicates with a small set of other tasks (its "neighbors"); in contrast, *global* communication requires each task to communicate with many tasks.
- In *structured* communication, a task and its neighbors form a regular structure, such as a tree or grid; in contrast, *unstructured* communication networks may be arbitrary graphs.

Definition

- In *static* communication, the identity of communication partners does not change over time; in contrast, the identity of communication partners in *dynamic* communication structures may be determined by data computed at runtime and may be highly variable.
- In *synchronous* communication, producers and consumers execute in a coordinated fashion, with producer/consumer pairs cooperating in data transfer operations; in contrast, *asynchronous* communication may require that a consumer obtain data without the cooperation of the producer.

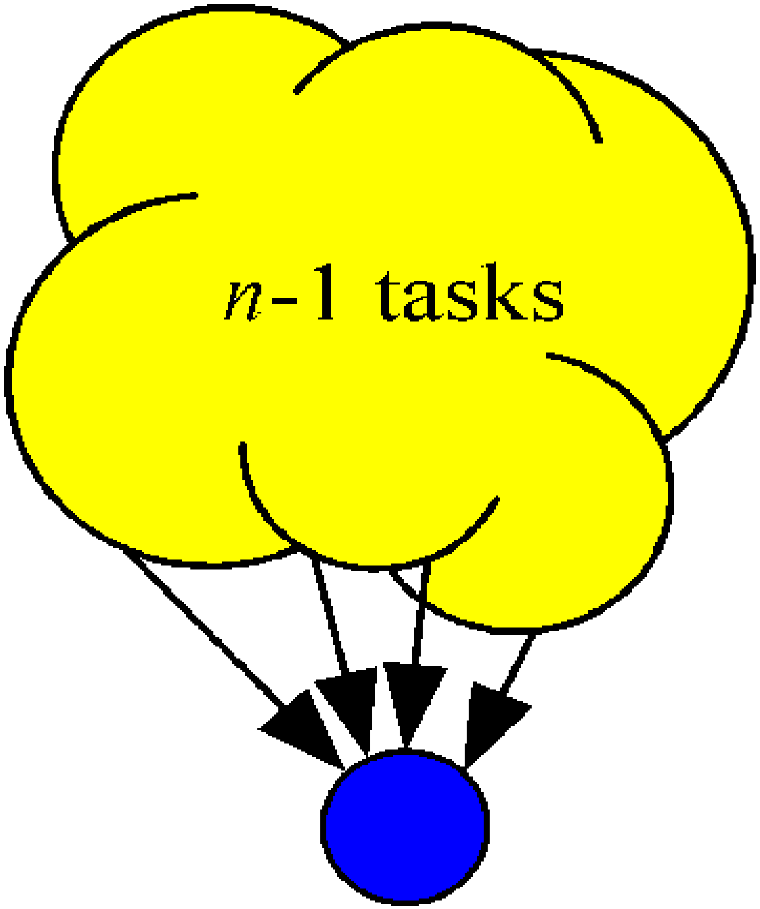
Decision tree to choose a mapping strategy


Static number of tasks
Dynamic number of tasks
Structured communication pattern
Unstructured communication pattern
Roughly constant computation time per task
Computation time per task varies
Frequent communications between tasks
Many short-lived tasks. No inter task communication
Join tasks to minimize communication. Create one task per processor
Cyclically map tasks to processors for computational load balancing
Use static load balancing techniques
Use dynamic load balancing techniques
Use run time task scheduling algorithms

Reduction

- Given associative operator $\oplus$
- $a_0 \oplus a_1 \oplus a_2 \oplus \ldots \oplus a_{n-1}$
- Examples
  - Add
  - Multiply
  - And, Or
  - Maximum, Minimum

Parallel Reduction Evolution

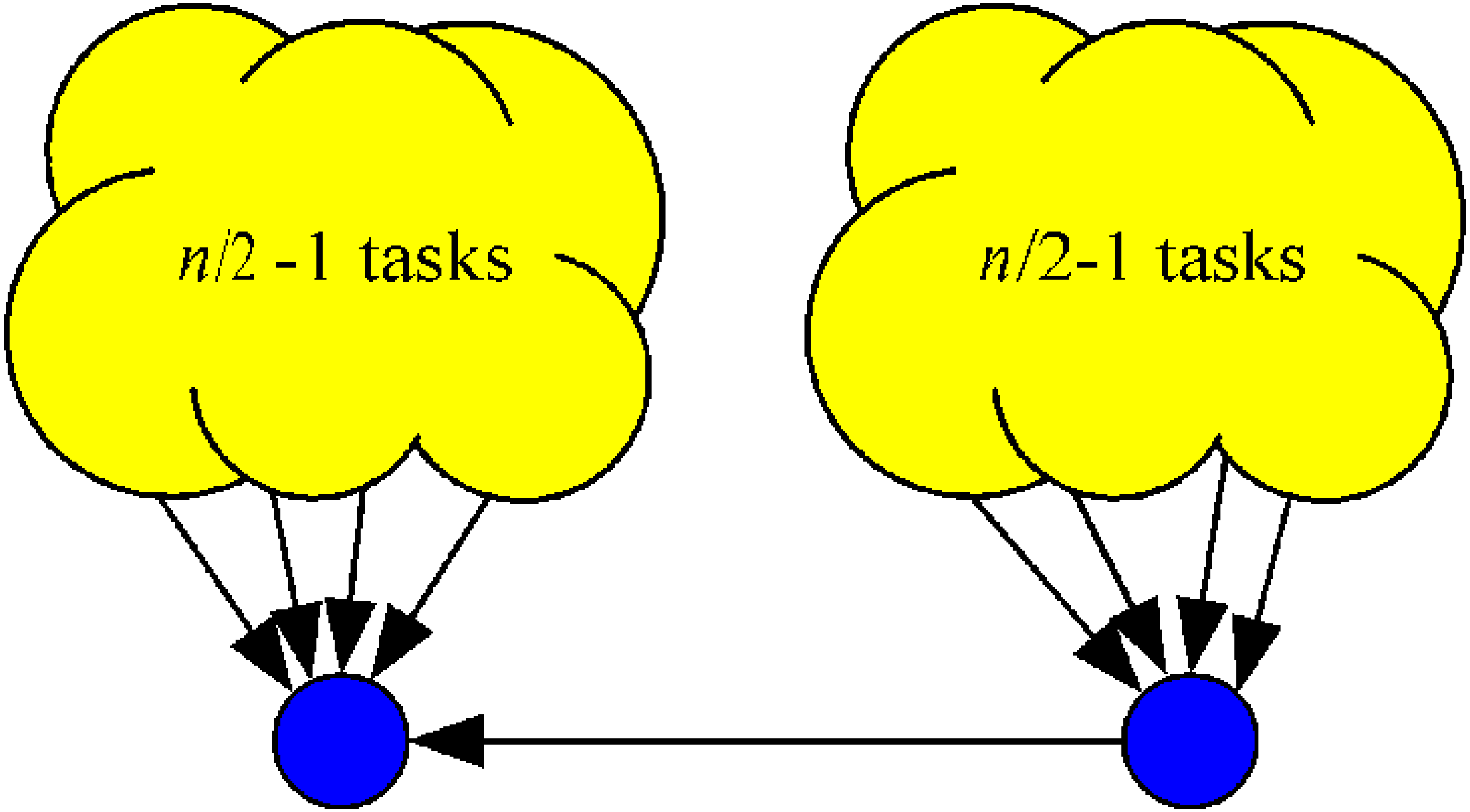


Observations:

1. If ***x*** time required for a task to communicate another task
2. ***y*** time is required for addition then
3. Total time will be (n-1)(x+y)

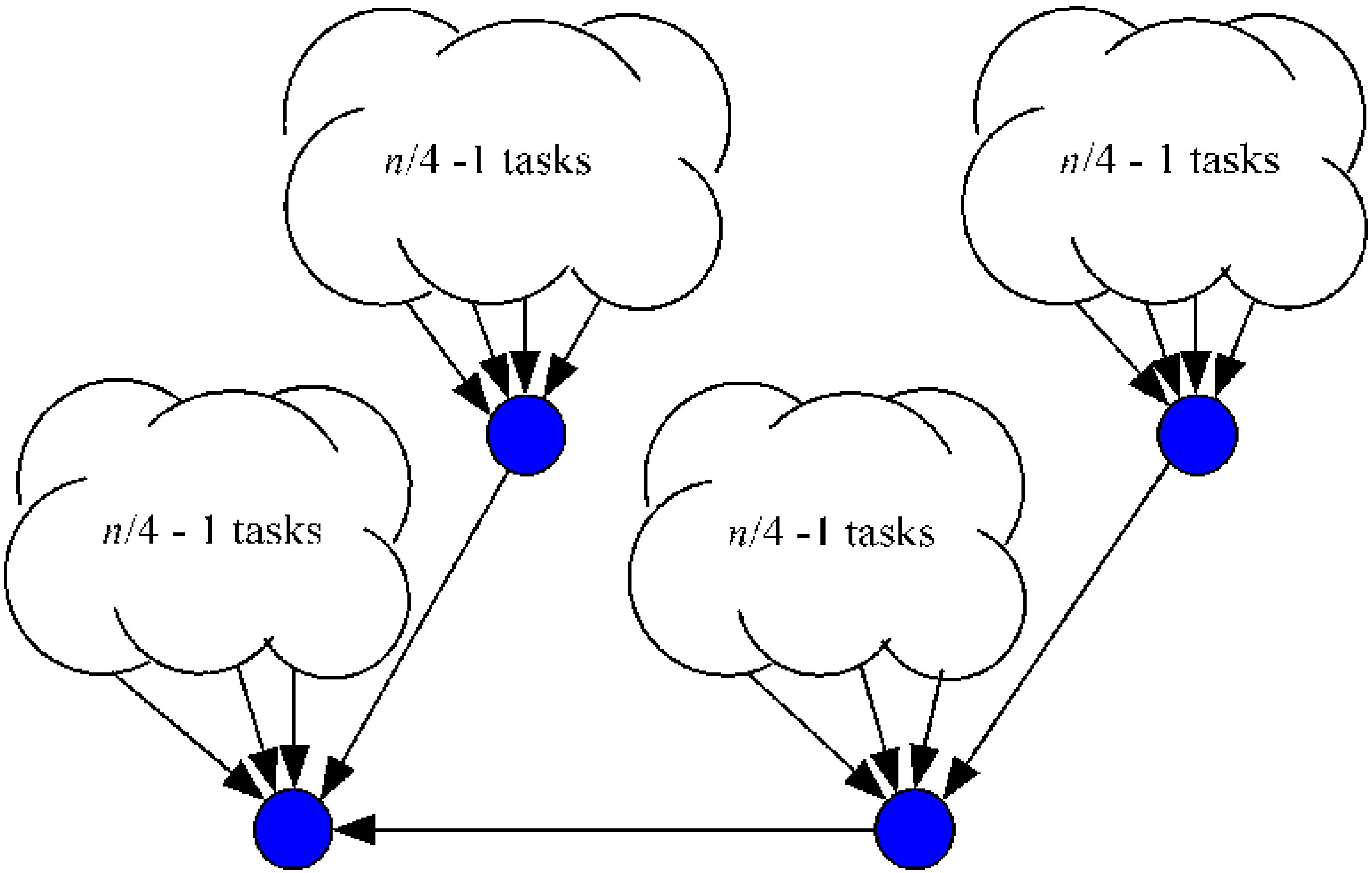
Source: M.J. Quinn

Parallel Reduction Evolution



Observations: Total time will be now (n/2-1)(x+y)

Parallel Reduction Evolution



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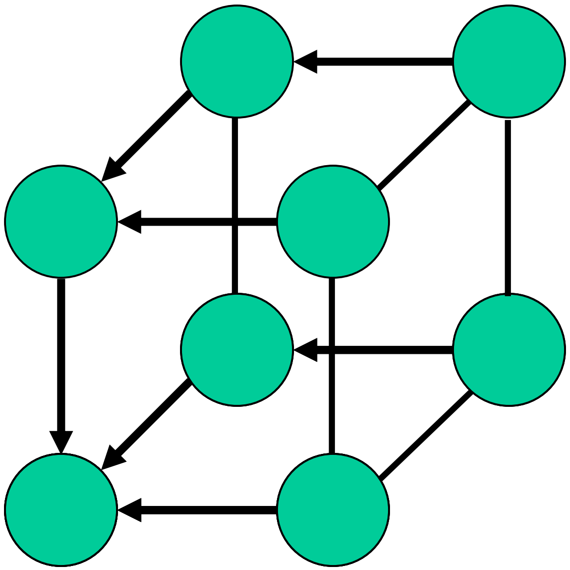
Binomial Trees

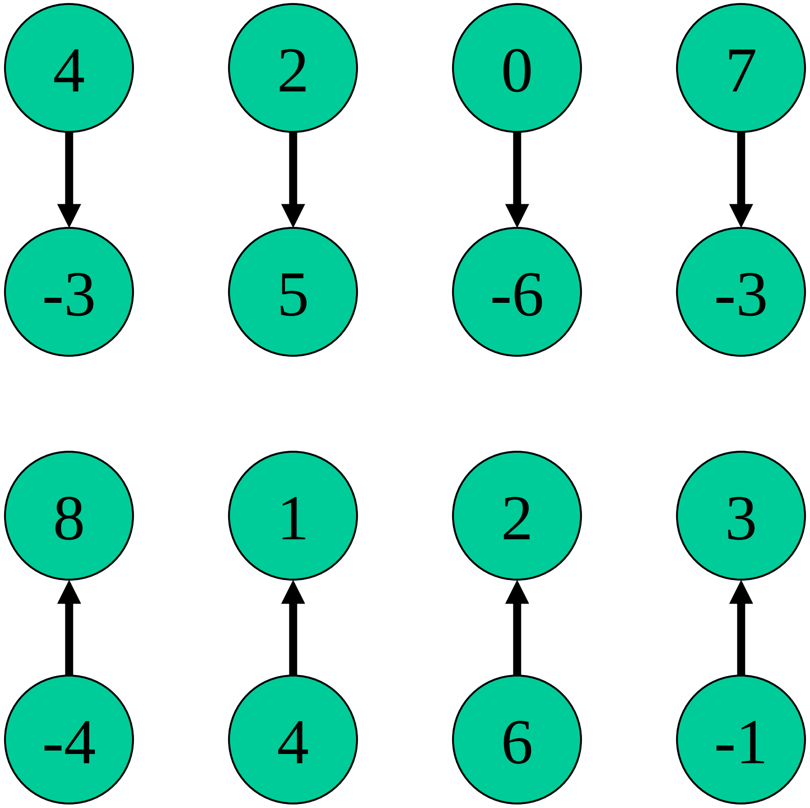


Subgraph of hypercube

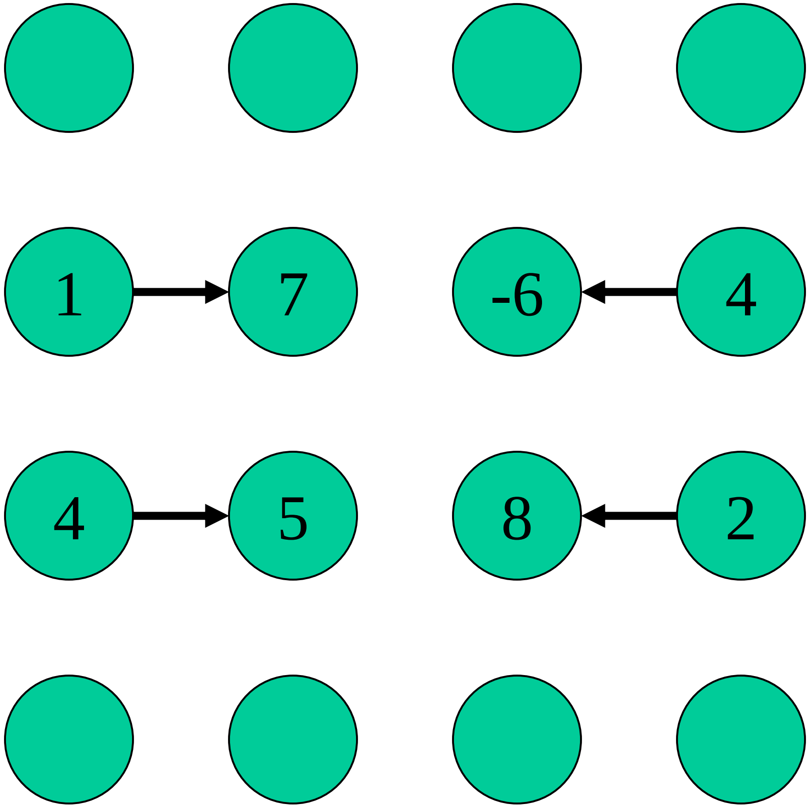
Observations:

Continuing this way we have n/2 semi-root tasks
Total time will be ***log n***

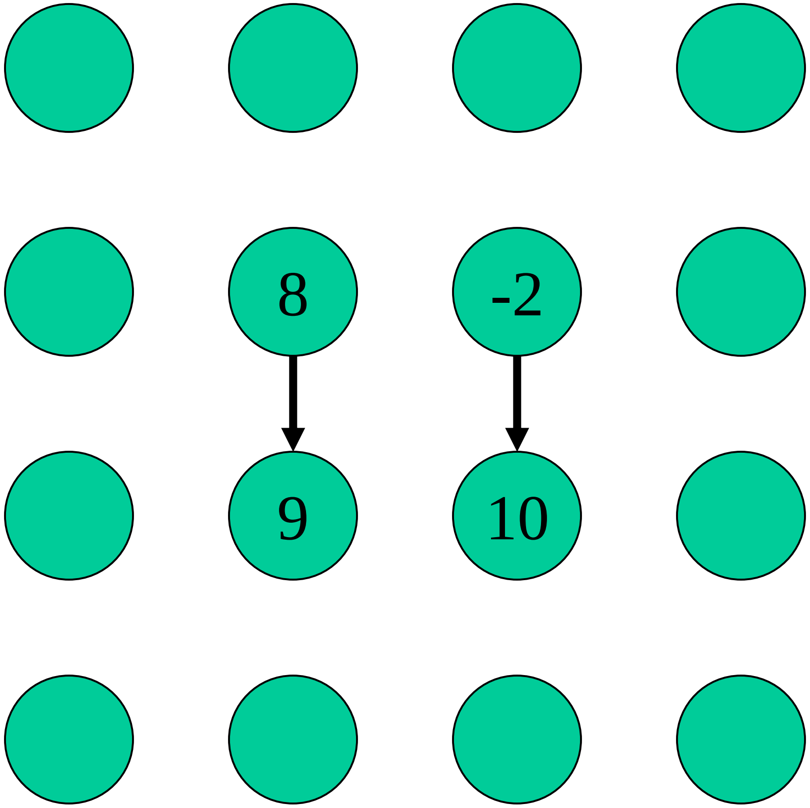
Finding Global Sum



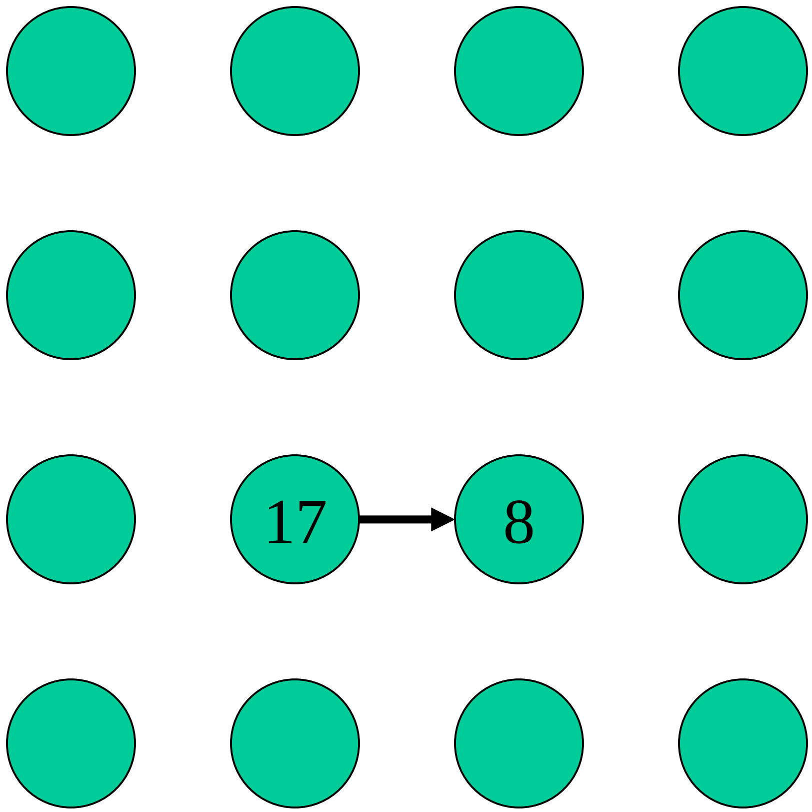
Finding Global Sum



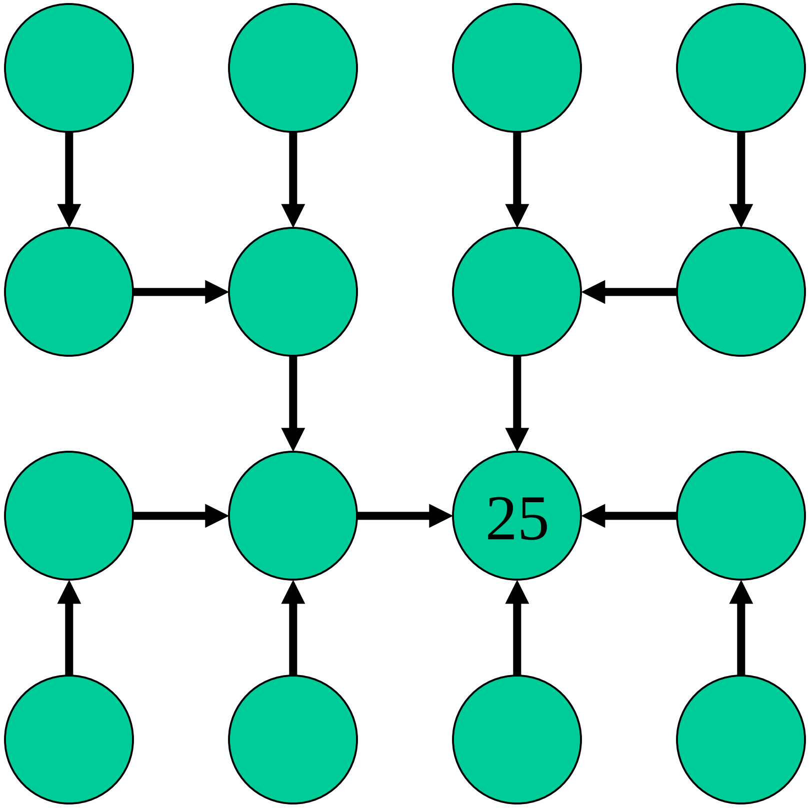
Finding Global Sum



Finding Global Sum

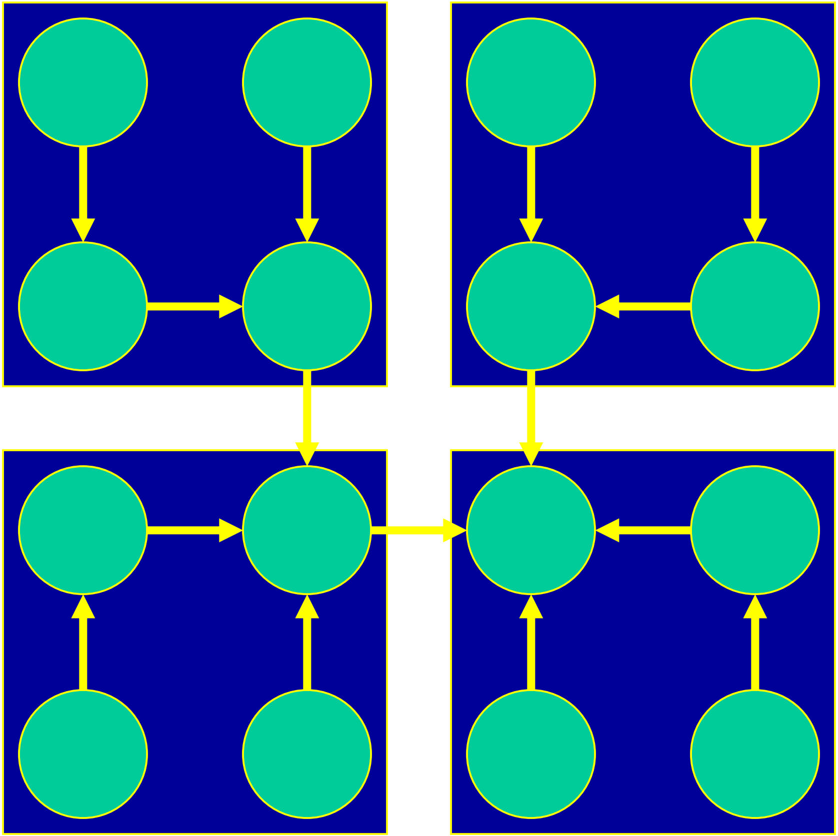


Finding Global Sum



Binomial Tree

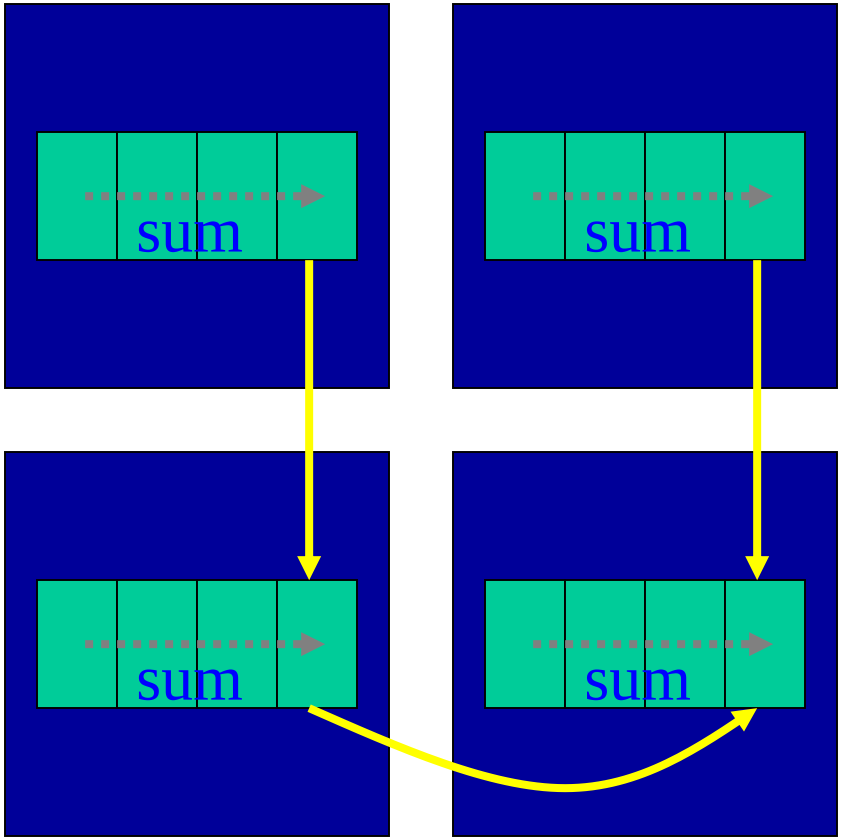
Agglomeration



Observations:

1. Number of tasks are static
2. Computations per task are trivial
3. Communication pattern is regular

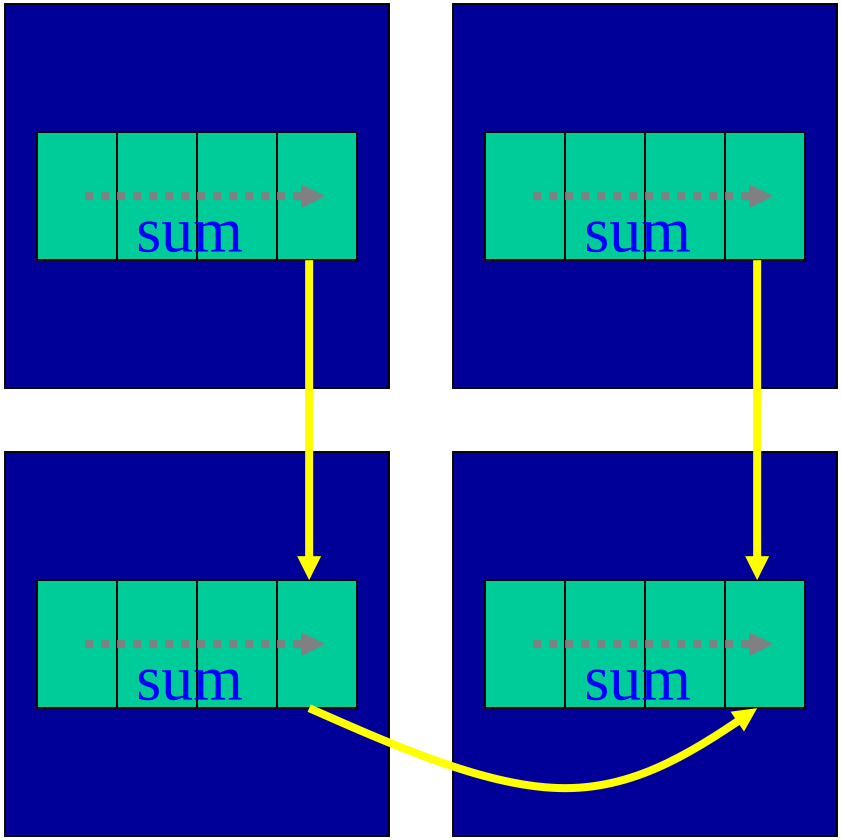
Agglomeration



Observations:

1. If ***x*** time required for a task to communicate another task
2. ***y*** time is required for binary operation then
3. Total time will be (n-1)(x+y)

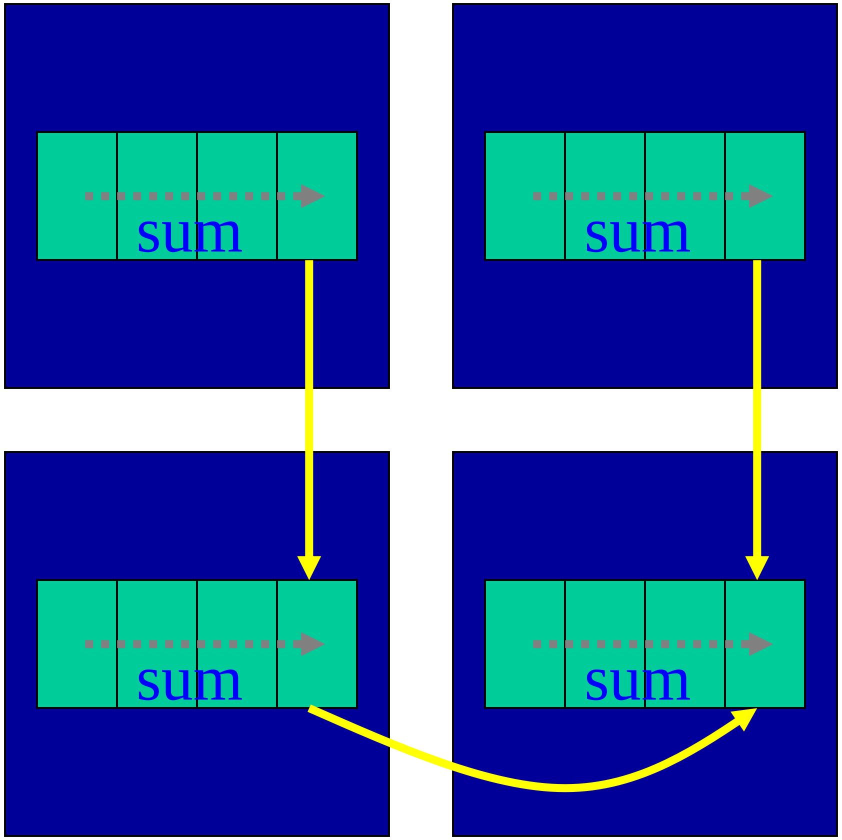
Agglomeration



Observations:

1. If we have ***p*** processors
2. Time require to compute sub total is $(\lceil n/p \rceil - 1)y$
3. Reduction require x+y time
4. With $\lceil \log p \rceil$ communications overall time

Agglomeration



Total time:

$$(\lceil n/p \rceil - 1)y + \lceil \log p \rceil (x+y)$$