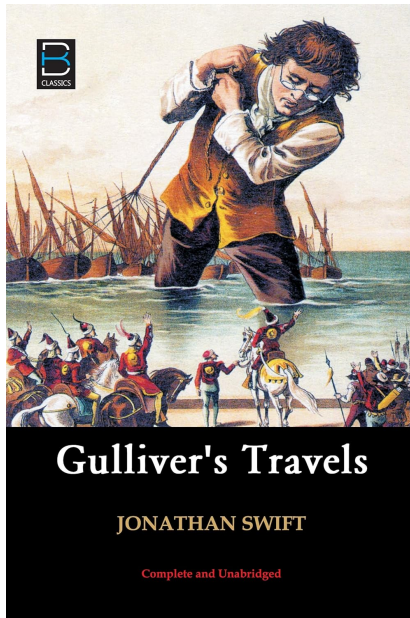


Gulliver's Travels: Meeting Giants and Lilliputians in the Number World

S.Durga Bhavani

School of Computer & Information Sciences,
University of Hyderabad.

19 December 2025



Acknowledgement: Google Images

What are the giant numbers that we know?

- Do we get large numbers with addition or multiplication?

What are the giant numbers that we know?

- Do we get large numbers with addition or multiplication?
- 10,

What are the giant numbers that we know?

- Do we get large numbers with addition or multiplication?
- 10, 100,

What are the giant numbers that we know?

- Do we get large numbers with addition or multiplication?
- 10, 100, 1000,

What are the giant numbers that we know?

- Do we get large numbers with addition or multiplication?
- 10, 100, 1000, 10000,

What are the giant numbers that we know?

- Do we get large numbers with addition or multiplication?
- 10, 100, 1000, 10000, 10^5 ,

What are the giant numbers that we know?

- Do we get large numbers with addition or multiplication?
- 10, 100, 1000, 10000, 10^5 , 10^6 ,
- What is the largest number you can think of?

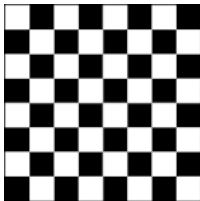
What are the giant numbers that we know?

- Do we get large numbers with addition or multiplication?
- 10, 100, 1000, 10000, 10^5 , 10^6 ,
- What is the largest number you can think of?
- The total atoms in the universe?

What are the giant numbers that we know?

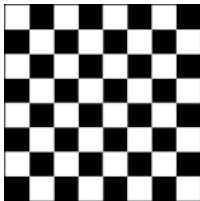
- Do we get large numbers with addition or multiplication?
- 10, 100, 1000, 10000, 10^5 , 10^6 ,
- What is the largest number you can think of?
- The total atoms in the universe? (supposed to be $\sim 10^{80}$)

Akbar Birbal story



Akbar used to play chess with Birbal almost every evening. One evening when Birbal won, Akbar was very pleased with Birbal's game and so offered him a grand reward.

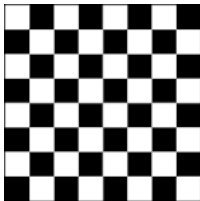
Akbar Birbal story



Akbar used to play chess with Birbal almost every evening. One evening when Birbal won, Akbar was very pleased with Birbal's game and so offered him a grand reward.

"Ask whatever you need. I am the king, I can grant you whatever you wish!"

Akbar Birbal story

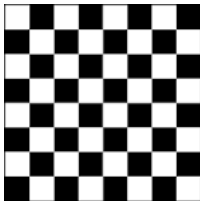


Akbar used to play chess with Birbal almost every evening. One evening when Birbal won, Akbar was very pleased with Birbal's game and so offered him a grand reward.

"Ask whatever you need. I am the king, I can grant you whatever you wish!"

Birbal was hesitant and then said,

Akbar Birbal story



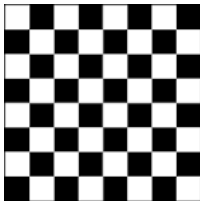
Akbar used to play chess with Birbal almost every evening. One evening when Birbal won, Akbar was very pleased with Birbal's game and so offered him a grand reward.

"Ask whatever you need. I am the king, I can grant you whatever you wish!"

Birbal was hesitant and then said,

"Raja!, my needs are simple: I would like to have one grain of wheat for the first square on the chessboard, two for second, four for third, and so on for all the 64 squares."

Akbar Birbal story



Akbar used to play chess with Birbal almost every evening. One evening when Birbal won, Akbar was very pleased with Birbal's game and so offered him a grand reward.

"Ask whatever you need. I am the king, I can grant you whatever you wish!"

Birbal was hesitant and then said,

"Raja!, my needs are simple: I would like to have one grain of wheat for the first square on the chessboard, two for second, four for third, and so on for all the 64 squares."

"Is that all?" Akbar was surprised. Birbal for once disappointed him.

Anyhow, Akbar commanded the bearers to get a sack of grain and start filling the chess board.

Anyhow, Akbar commanded the bearers to get a sack of grain and start filling the chess board.

What do you think was the size of Birbal's sack?

Anyhow, Akbar commanded the bearers to get a sack of grain and start filling the chess board.

What do you think was the size of Birbal's sack?

$$1 + 2 + 2^2 + 2^3 + \dots + 2^{63}$$

Anyhow, Akbar commanded the bearers to get a sack of grain and start filling the chess board.

What do you think was the size of Birbal's sack?

$$1 + 2 + 2^2 + 2^3 + \dots + 2^{63}$$

$$= 2^{64} - 1$$

Anyhow, Akbar commanded the bearers to get a sack of grain and start filling the chess board.

What do you think was the size of Birbal's sack?

$$1 + 2 + 2^2 + 2^3 + \dots + 2^{63}$$

$$= 2^{64} - 1$$

$$18446744073709551616 = 1.84 \times 10^{19}$$

An exponential function!

Anyhow, Akbar commanded the bearers to get a sack of grain and start filling the chess board.

What do you think was the size of Birbal's sack?

$$1 + 2 + 2^2 + 2^3 + \dots + 2^{63}$$

$$= 2^{64} - 1$$

$$18446744073709551616 = 1.84 \times 10^{19}$$

An exponential function!

As they started filling the grains as per the chess-board, soon the granary was getting empty!!

Anyhow, Akbar commanded the bearers to get a sack of grain and start filling the chess board.

What do you think was the size of Birbal's sack?

$$1 + 2 + 2^2 + 2^3 + \dots + 2^{63}$$

$$= 2^{64} - 1$$

$$18446744073709551616 = 1.84 \times 10^{19}$$

An exponential function!

As they started filling the grains as per the chess-board, soon the granary was getting empty!!

Akbar realized that he spoke out of arrogance as a king and was aptly answered by Birbal!

Time Complexity of an Algorithm

To understand the time taken by an algorithm, it is expressed as a function of n , the input size.

For example, what is the time taken by the Google Search algorithm?

How big is 2^n ?

Function/ $n \rightarrow$	10	50	100	1000
n^2	100	2500	10,000	10^6

How big is 2^n ?

Function/ $n \rightarrow$	10	50	100	1000
n^2	100	2500	10,000	10^6
n^3	1000	125,000	10^6	1 billion

How big is 2^n ?

Function/ $n \rightarrow$	10	50	100	1000
n^2	100	2500	10,000	10^6
n^3	1000	125,000	10^6	1 billion
2^n				

How big is 2^n ?

Function/ $n \rightarrow$	10	50	100	1000
n^2	100	2500	10,000	10^6
n^3	1000	125,000	10^6	1 billion
2^n	1024			

How big is 2^n ?

Function/ $n \rightarrow$	10	50	100	1000
n^2	100	2500	10,000	10^6
n^3	1000	125,000	10^6	1 billion
2^n	1024	16 digits	31 digits	302 digits

How big is 2^n ?

Function/ $n \rightarrow$	10	50	100	1000
n^2	100	2500	10,000	10^6
n^3	1000	125,000	10^6	1 billion
2^n	1024	16 digits	31 digits	302 digits
$n!$	3.6 million	65 digits	161 digits	very large

Table: Courtesy: Harel: Algorithmics (The number of protons in the universe has 79 digits)

Puzzle!

Make the largest number you can using four 2's.

- What is the smallest? $2 + 2 + 2 + 2$?

Puzzle!

Make the largest number you can using four 2's.

- What is the smallest? $2 + 2 + 2 + 2$?
- Any other answer?

Puzzle!

Make the largest number you can using four 2's.

- What is the smallest? $2 + 2 + 2 + 2$?
- Any other answer? we will come back to this later.
- Now for large numbers!
- 2222,

Puzzle!

Make the largest number you can using four 2's.

- What is the smallest? $2 + 2 + 2 + 2$?
- Any other answer? we will come back to this later.
- Now for large numbers!
- 2222, 222^2 ,

Puzzle!

Make the largest number you can using four 2's.

- What is the smallest? $2 + 2 + 2 + 2$?
- Any other answer? we will come back to this later.
- Now for large numbers!
- 2222 , 222^2 , 22^{22} ,

Puzzle!

Make the largest number you can using four 2's.

- What is the smallest? $2 + 2 + 2 + 2$?
- Any other answer? we will come back to this later.
- Now for large numbers!
- 2222 , 222^2 , 22^{22} , 2^{222}
- How do we know which is larger? Let us use some simple arithmetic.

Puzzle!

- How do we know which is larger: 222^2 , 22^{22} ?
Multiply 222×222 and tell me.

Puzzle!

- How do we know which is larger: 222^2 , 22^{22} ?
Multiply 222×222 and tell me. it is 49284 (just 5 digits).
- What about 22^{22} ? How many digits will it have?

Puzzle!

- How do we know which is larger: 222^2 , 22^{22} ?
Multiply 222×222 and tell me. it is 49284 (just 5 digits).
- What about 22^{22} ? How many digits will it have?

$$22^{22} > 20^{22} > 10^{22}$$

Puzzle!

- How do we know which is larger: 222^2 , 22^{22} ?
Multiply 222×222 and tell me. it is 49284 (just 5 digits).
- What about 22^{22} ? How many digits will it have?

$$22^{22} > 20^{22} > 10^{22}$$

- Clearly, 10^2 has 2 zeros, 10^3 has 3 zeros, and so on, 10^{22} has 22 zeros. It is a 23 digit number. So 22^{22} is atleast a 23 digit number!!
- Therefore, 22^{22} is much much greater than 222^2 .

Puzzle!

- How do we know which is larger: 222^2 , 22^{22} ?
Multiply 222×222 and tell me. it is 49284 (just 5 digits).
- What about 22^{22} ? How many digits will it have?

$$22^{22} > 20^{22} > 10^{22}$$

- Clearly, 10^2 has 2 zeros, 10^3 has 3 zeros, and so on, 10^{22} has 22 zeros. It is a 23 digit number. So 22^{22} is atleast a 23 digit number!!
- Therefore, 22^{22} is much much greater than 222^2 .
- What about 22^{22} and 2^{222} ?

Puzzle!

- How do we know which is larger: 222^2 , 22^{22} ?
Multiply 222×222 and tell me. it is 49284 (just 5 digits).
- What about 22^{22} ? How many digits will it have?

$$22^{22} > 20^{22} > 10^{22}$$

- Clearly, 10^2 has 2 zeros, 10^3 has 3 zeros, and so on, 10^{22} has 22 zeros. It is a 23 digit number. So 22^{22} is atleast a 23 digit number!!
- Therefore, 22^{22} is much much greater than 222^2 .
- What about 22^{22} and 2^{222} ? Try!
-

$$22^{22} < 32^{22} = (2^5)^{22} =$$

Puzzle!

- How do we know which is larger: 222^2 , 22^{22} ?
Multiply 222×222 and tell me. it is 49284 (just 5 digits).
- What about 22^{22} ? How many digits will it have?

$$22^{22} > 20^{22} > 10^{22}$$

- Clearly, 10^2 has 2 zeros, 10^3 has 3 zeros, and so on, 10^{22} has 22 zeros. It is a 23 digit number. So 22^{22} is atleast a 23 digit number!!
- Therefore, 22^{22} is much much greater than 222^2 .
- What about 22^{22} and 2^{222} ? Try!
-

$$22^{22} < 32^{22} = (2^5)^{22} = 2^{110} < 2^{222}$$

- So is 2^{222} the largest number we can think of? Try some more!

Puzzle: The power of powers!

- How about $2^{2^{2^2}}$?

Puzzle: The power of powers!

- How about $2^{2^{2^2}}$? That is, $2^{484} > 2^{222}!!!$

Puzzle: The power of powers!

- How about $2^{2^{2^2}}$? That is, $2^{484} > 2^{222}!!!$
Is this the biggest??

Puzzle: The power of powers!

- How about $2^{2^{2^2}}$? That is, $2^{484} > 2^{222}!!!$
Is this the biggest??
- $2^{2^{2^2}}$ is the largest!! This number is supposed to have more than 10 lakh digits!

Logarithms

What are Logarithms?

These are complement of exponential!

Logarithms

What are Logarithms?

These are complement of exponential!

- **Definition:** If $2^n = a$ then $n = \log_2(a)$ (We will use base 2 in our discussion).

Logarithms

What are Logarithms?

These are complement of exponential!

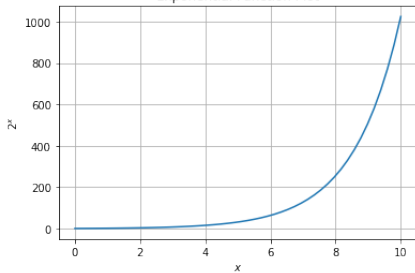
- **Definition:** If $2^n = a$ then $n = \log_2(a)$ (We will use base 2 in our discussion).
- That is, if a giant has walked a length of 2^n , the number of steps it has taken is $\log(2^n)$ which is n .

What are Logarithms?

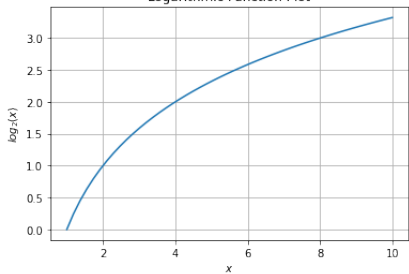
These are complement of exponential!

- **Definition:** If $2^n = a$ then $n = \log_2(a)$ (We will use base 2 in our discussion).
- That is, if a giant has walked a length of 2^n , the number of steps it has taken is $\log(2^n)$ which is n .
- Hence $\log n$ is very very slowly increasing function.

Exponential Function Plot

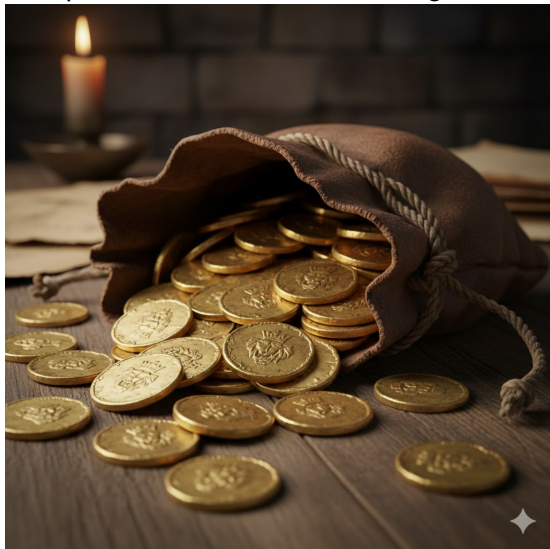


Logarithmic Function Plot



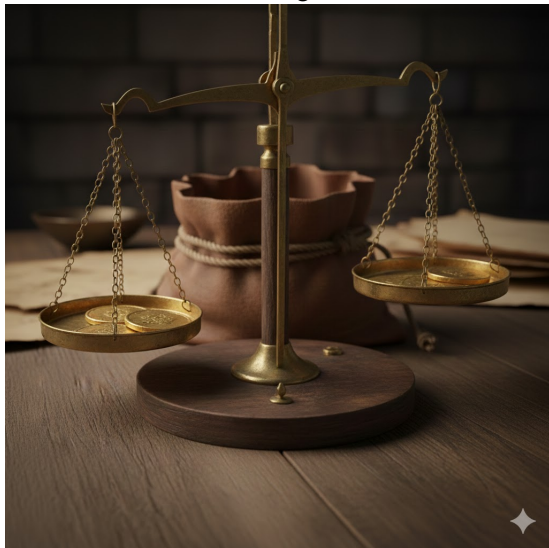
Puzzle!

There are 100 gold coins in a bag, all of the same weight except one which is fake and is lighter.

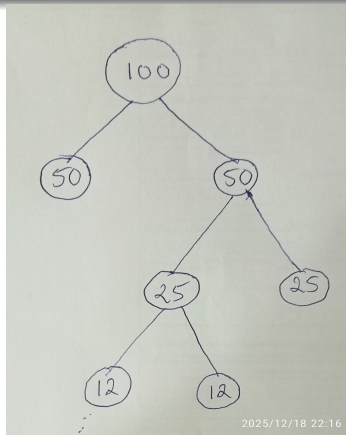


Puzzle contd..

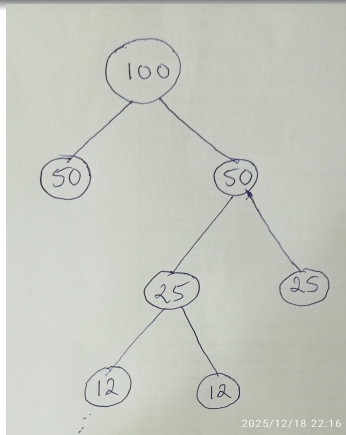
Find the fake coin using minimum number of weighings.



What is the answer?

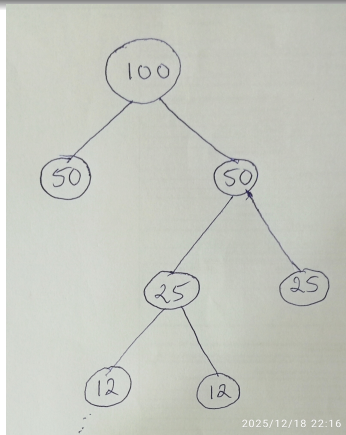


What is the answer?



$100 \rightarrow 50 \rightarrow 25 \rightarrow 12 \rightarrow 6 \rightarrow 3 \rightarrow 1$

What is the answer?



$100 \rightarrow 50 \rightarrow 25 \rightarrow 12 \rightarrow 6 \rightarrow 3 \rightarrow 1$

$$\log_2 100 \sim 6.647$$

Binary Search

- You think of a number between 1 and 100. And I want to guess it.

Binary Search

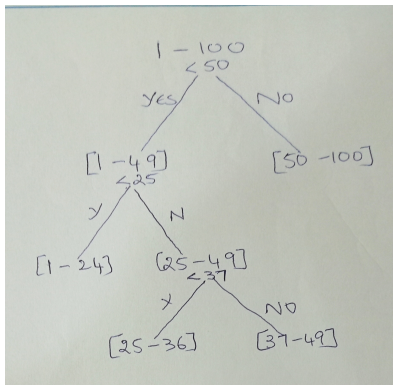
- You think of a number between 1 and 100. And I want to guess it.
- I can ask any question, but you will answer only YES or NO.

Binary Search

- You think of a number between 1 and 100. And I want to guess it.
- I can ask any question, but you will answer only YES or NO.
- How many questions should I ask before I can find the number in your mind.

Binary Search

- You think of a number between 1 and 100. And I want to guess it.
- I can ask any question, but you will answer only YES or NO.
- How many questions should I ask before I can find the number in your mind.



The Powers and the Logarithms

Function/ $n \rightarrow$	10	50	100	1000
$\log n$	1	1.7	2	3
$n \log_2 n$	33	282	665	9966
n^2	100	2500	10,000	10^6
n^3	1000	125,000	10^6	1 billion
2^n	1024	16 digits	31 digits	302 digits
$n!$	3.6 million	65 digits	161 digits	very large

Table: Courtesy: Harel: Algorithmics (The number of protons in the universe has 79 digits)