

Math III
ASSIGNMENT - 2
SUBMIT by 26th September 2016

1. The equation $x^2y'' + pxy' + qy = 0$, where p and q are constants, is called Euler's equi-dimensional equation. Show that the change of independent variable given by $x = e^z$ transforms it into an equation with constant coefficients. Apply this technique to find the general solution of each of the following equations:

(a) $x^2y'' + 3xy' + 10y = 0$

(b) $2x^2y'' + 10xy' + 8y = 0$

(c) $x^2y'' + 2xy' - 12y = 0$;

2. Find the general solution of each of the following equations

(a) $y'' + 10y' + 25y = 14e^{-5x}$

(b) $y'' - 2y' + 5y = 25x^2 + 12$;

3. If $y_1(x)$ and $y_2(x)$ are particular solutions of $y'' + P(x)y' + Q(x)y = R_1(x)$ and $y'' + P(x)y' + Q(x)y = R_2(x)$, show that $y(x) = y_1(x) + y_2(x)$ is a solution of $y'' + P(x)y' + Q(x)y = R_1(x) + R_2(x)$.

Use this principle to find the general solution of

$$y'' + 4y = 4\cos 2x + 6\cos x + 8x^2 - 4x$$

4. Find a particular solution of each of the following equations:

(a) $y'' + 2y' + y = e^{-x}\log x$;

(b) $y'' + 2y' + 5y = e^{-x}\sec 2x$

5. Find the general solution of the following equation:

$$(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2$$