Algorithms Group Assignment

October 28, 2019

- You need to think of design paradigms that suit the problem as well as appropriate data structures that make the implementation efficient.
- Think of at least two different ways of solving the problem (mostly, greedy and other).
- Prepare a presentation that contains: a) Problem statement b) Motivation c) Algorithms d) Good to have a tracing of your algorithm on a toy example.

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1. Auto-word completion problem: Design algorithms that output top browsed phrases, on a partial query word (phrase) that complete the word (phrase).
2. You have n elements $\{a_1, a_2, \ldots, a_n\}$ that need to be connected using relations $<$ and $=$. Give a dynamic programming algorithm that can calculate, as a function of m, the number of different possible orderings. For example, $n = 3$ gives 13 orderings, with $a_1 < a_2 < a_3$, $a_1 < a_2 = a_3$, $a_1 = a_2 < a_3$, $a_1 = a_2 = a_3$, $a_2 < a_1 < a_3$ and so on.
3. You are designing a house in an empty rectangular plot of dimensions $m \times n$ given requirements of the number of rooms etc. Plan an algorithmic approach to the design so that you can get different feasible plans that satisfy the requirements.
4. Design an $O(n)$ algoithm that finds all the elements that appear more than $n/2$ times in a list of size n. Then, design an $O(n)$ algoithm that finds all the elements that appear more than $n/4$ times in the list.
5. If you are a cab service owner, people appear on the map and start demanding rides, in a rush-hour scenario. Design algorithms that will dispatch your fleet of vehicles in a way that can maximize your profits. In essence: more passengers = more money.
6. Let us imagine that Dijkstra algorithm is being used by a cab services. (i) Explain how to provide an alternate route when the shortest route has a traffic jam. (ii) How do the rides get assigned to the drivers if the criteria are longest waiting driver should be given a priority all other conditions being equal

7. Consider the alphabet $\Sigma = \{a, b, c\}$. The elements of Σ have the following multiplication table : For example, read the first row as $a.a = b; a.b = b; a.c = a$. Find an efficient algorithm that decides for given string x if there exists a parenthesization that evalates to 'a' as an answer. (For eg if $x = bbbba$, it returns yes as $(b(bb))(ba) = a$.

	a	b	c
a	b	b	a
b	c	b	a
c	a	c	c

8. Let P be a simple, but not necessarily convex polygon and x an arbitrary point (not necessarily in P. Design an efficient algorithm to find a line segment originating from x that interesects the maximum number of edges of P.

9. Graph Visualization: Propose an algorithm that draws a rooted tree of with the number of branches given at each node such that no two edges cross.

10. Given a set S of n integers and an integer W, give an algorithm to test whether k of the integers in S add upto W.

11. DATA DE-DUPLICATION: De duplication is a technique used to store repeated data only once. This data might be spread across multiple files or for same file with multiple versions. Assume that you have a Pattern_Matcher, that is a pattern matching algorithm which given a pattern 'P' returns all the files that contain P as well as the location (grep -H will do!). Design a data de-duplication algorithm with appropriate data structures if necessary.

12. You have a computer with 2 MB of main memory how do you use it to sort a large file of 500 MB that is on the disk ? Propose an algorithm for this task.

13. A set of books of different sizes(number of pages), $S_1, S_2, \ldots, S_n$ and integer K is given. Propose an efficient algorithm to partition S into K groups such that the maximum size of any group is minimized. You can not reorder the arrangement of books and size of a group is equal to the sum of the sizes of the books in the group.

14. You are given an array of N numbers which can be positive, negative or 0. Give an efficient algorithm to identify the index postions i and j such that the sum of the numbers from i to j is maximun.

15. We have a table of M binary strings, called code words each atmost k in length. We want to encode an input string S of length n using as few binary strings as possible. Given an $O(nmk)$ algorithm to find the length

of the best encoding. For example, if our code words are (a, ba, bab) then the string *babababa* can be encoded using 4 code words.

16. Given a network of cities modeled as a tree, the problem is to parition the network into two groups A and B such that a group cannot attack the other group. A city i of group, say, A can attack a city of group j of group B if j is a neighbour of i and j does not have any neighbours of its own group B. Find the number of ways of partitioning the tree into two sets of nodes such that one group cannot attack the other. (Trivial partition is that of taking all the nodes into one group)

17. Given a set of n students and a set of n problems and students give a preference integer rating in the range $\{0,\ 10\}$ (0 indicates the least preference) propose an algorithm that assigns one problem to each student such that the assignment maximizes the 'happiness' of the students.

18. SUDOKU Consider a variation in Sudoku where the diagonal and anti-diagonal of the entire square also are constrained to have distinct numbers. Propose an algorithm to solve this puzzle.

19. Suppose we have $A = \{a_1, a_2, \ldots a_n\}$ positive integers and given p we can form p sums of from a by taking p numbers from A (repetitions allowed) as $(a_{i_1} + a_{i_2} + \ldots a_{i_p}$. In the sums repetition of numbers is allowed.

    Now the problem to be solved is if the sequence of sums and p are given, to determine the original set of numbers from which the p sums are derived.

    For example, if we are given $\{6, 7, 2, 4, 5, 8\}$ and $p = 2$ we can derive the original set to be $\{1, 3, 4\}$.

    Propose an algorithm to solve this problem.

20. Devise an algorithm to find the number of ways to construct a sequence of length n from an array of numbers A such that consecutive positions contain distinct numbers and the first and last positions are the same. The numbers within a sequence may be repeated.

    For example, the first position is given as 1 and the last as 1 and suppose $A = \{1, 2, 3, 4\}$ and $n = 4$. Then the possible number of sequences is $3 \times 6 = 18$. (one set of choices is:

    1, 2, 3, 1
    1, 3, 2, 1
    1, 2, 4, 1
    1, 4, 2, 1
    1, 3, 4, 1
    1, 4, 3, 1

    Please note that if $n = 5$ then $1, 2, 1, 3, 1$ is also possible.