University of Hyderabad
Department of Computer & Information Sciences
MTECH(AI) - ALGORITHMS - MAJOR EXAMINATION

20 November 2009 Total Marks: 60 Duration: 3 hours

Read each question carefully.
Good time management is necessary;
Do not spend more than 15 minutes on any questions of Part A.

PART A : Answer any 6 questions. Each question carries 4 marks

1. Partition the following functions by their asymptotic growth into equivalence classes such that $f(n)$ and $g(n)$ are equivalent if $f(n) = \Theta(g(n))$.

$$2n, 3, nlgn, lg\sqrt{(}n), 4^{lgn}, e^n, lg(n!), (\frac{3}{2})^n$$

2. You are given an array A of n real numbers sorted in nondecreasing order and value v is given. Write an $O(n)$ algorithm that returns true if there are distinct indexes i and j such that $A[i] + A[j] = v$ and false otherwise and prove that the time complexity is $O(n)$.

3. Give a counter example to show that the following greedy strategy does not give an optimal solution for chained matrix multiplication problem: The product $(M_1 M_2 \dots M_n)$ is grouped as $(M_1 M_2 \dots M_k)(M_{k+1} \dots M_n)$ where M_k has minimum number of columns, $1 \leq k \leq n$.

4. Define the complexity classes **NP** and **Co-NP** and give an example for each of the classes. Explain.

5. Show that the problem of finding Hamiltonian Path in a directed acyclic graph belongs to the class P.

6. Derive the charaterizing relations for **forward and backward** edges present in the DFS tree (obtained after a DFS traversal of a directed graph), in terms of discovery 'd'and finishing 'f' times for nodes. (For example (u, v) is a cross edge if and only if $d(v) < f(v) < d(u) < f(u)$.)

7. Construct optimal Huffman coding tree for the set of characters whose frequency of occurence is given as: A - 16; B - 22; C - 60; D - 4; E - 6; F - 13. Show all the intermediate steps of construction.

PART - B : Answer all the 4 questions. Note that there is some internal choice. Each question carries 9 Marks

8. A State Master's theorem for recurrence relations and apply it to find the solution of the following equation in Θ notation:

$$T(n) = 9T(n/2) + n^3, n \geq 2$$

B Express the number of times that the statement $x = x + 1$ gets executed in the following code segement using Θ notation:

```
j = n
While (j > = 1) {
        for i = 1 to j
        x = x+1
    j = j/2
}
```

9. A Describe an algorithm that computes articulation points in an undirected graph using DFS method. Trace it on an example.

B Explain why the rule highest(x) $\leq$ highest (v) does not characterize an articulation point v where x is a child node of v.

10. Solve the following **0/n** knapsack problem: The knapsack to be filled is of capacity 12 units with five items of weights 2, 3, 4, 5, and 6 and respective profits of 6, 8, 13, 16, 18 using **Branch and Bound Strategy**. What is the best case time complexity of this solution. Explain.

OR

Solve the Traveling salesperson problem that finds a minimal Hamiltonian circuit at 'a' using **dynamic programming strategy** on the following weighted graph that is given as the adjacency matrix. Clearly specify the subproblems and state the optimality criterion that is present. Analyze the time complexity of your solution.

	a	b	c	d
a	0	15	10	12
b	8	0	10	9
c	12	8	0	12
d	13	7	9	0

11. A Prove that if a problem A is NP Complete and $A \leq_P B$, $B \in NP$, then B is NP-Complete.

B Describe the Vertex Cover problem. Prove that Vertex Cover problem is NP-Complete by taking another NP-Complete problem for your use.

OR

A Explain what the expression $2SAT \in P$? means. Prove that the expression is true.

B Prove that problems D-HAM and Compute-HamCycle are polynomially equivalent.

GOOD LUCK