CHAPTER

1

THE PHYSICS OF COLOUR

Colour is one of the primary sources of information for human beings. Yet, it is very difficult to define colour precisely. Philosophers and scientists debated colour definition for centuries while Goethe stated strongly that colour is an experience and should not be modelled mathematically. He emphasised the physiological and psychological aspects of colours and went to the extent of saying that, "the colour theory is Newton's greatest error." In the middle of all the seeming chaos, scientists provided a simple definition which is good enough for us!

> "Colour is the response of the human vision system, comprising the eye-brain combination, to external stimuli in the form of electromagnetic radiation within a certain narrow range of wavelengths."

The definition is precise and captures several interesting aspects of colour. The first, it is entirely human. We say that a banana is yellow and an apple is red but an alien may not even see that they have different colours although the difference in shape may still be perceived. Second, colour is not merely related to the eye but to the brain as well. Today, it is understood that the signals sent from the eye via the optic nerve are *interpreted* by the brain as colour. Third, the input to colour perception is electromagnetic radiation or energy. We perceive electromagnetic energy only if its wavelength lies between approximately 4×10^{-7} m to 7×10^{-7} m. It is more common to express it as $400 - 700$ nanometres (nm) or as $0.4 - 0.7$ microns. This range is called, quite appropriately, the *visible spectrum* (see Figure 1.1).

It is apparent that our vision system is quite narrow and sensitive only to a very small part of the entire spectrum. There is so much more to see and discover! We use instruments to *see* and uncover information from these otherwise invisible regions of the electromagnetic spectrum. Radio telescopes detect long wavelengths and showed the presence and structure of galaxies with unprecedented details; radio waves are also used to listen to audio broadcasts from around the world. Radar

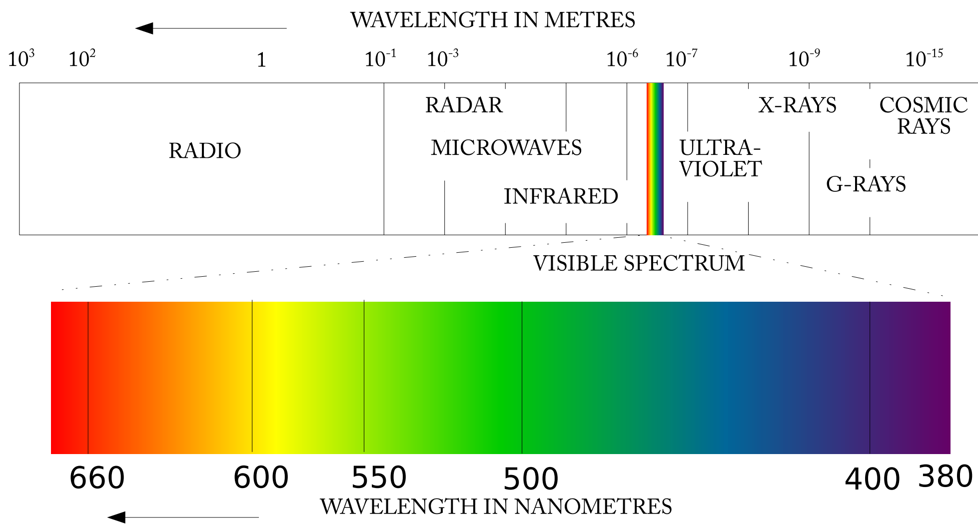

Figure 1.1: *The entire electromagnetic spectrum (top) and the visible spectrum (bottom)*

allows us to penetrate clouds and dust for viewing objects under poor seeing conditions; microwave lasers are used to measure distances accurately; infrared, ultraviolet and X-rays find uses in medical field; gamma rays and cosmic rays are observed with specially designed instruments in astronomy to detect and understand some of the most violent events in the universe. Thus, instruments allow us to perceive almost the entire electromagnetic spectrum even though we can sense only a tiny part of it directly.

Physics provides the simplest view of colour: when electromagnetic radiation of a specific wavelength between 380 nm and 700 nm falls on the eye, the brain interprets it as a specific colour. For example, a wavelength of 450 nm would be perceived as the colour blue and a wavelength of 660 nm is seen as red. If the energy falling on the eye-brain system consists of a single wavelength, then it is defined as a *pure* colour or a *monochromatic* colour. Impure colours result from a mixture of wavelengths. The exact nature of colour vision is as much a property of the human vision system as that of physics. Human vision is highly non-linear and the transformation of a spectrum into a perceived colour is a complex process.

A majority of our understanding of colour has been obtained by treating it as an optical phenomenon, i.e. as an interaction between light and matter. More importantly, colour science is relatively less well-known in the image processing community than radiometry that deals exclusively with brightness and hence relevant to grayscale processing. Colour science is a combination of physics, neurophysiology and empiricism. It is necessary to study the physics of colour if we were to process colour images *meaningfully in ways that mimic the physical processes occurring in nature* to produce realistic images.

1.1 Spectral Power Distribution Functions

Energy falling on the eye is almost always a combination of several wavelengths. That is, energy is distributed among different wavelengths and may be expressed as a function

$$E = S(\lambda)$$

$S(\lambda)$ is called the *Spectral Power Distribution Function (SPDF)* and defines how energy content varies with wavelength λ.

Two typical SPDFs are shown in Figure 1.2. The first SPDF, in (a), contains more energy in the shorter wavelengths between 450 nm and 500 nm and appears blue to the eye. The spectrum is that of the Munsell colour 10B 5/8. The corresponding colour is shown in the rectangle inset in the figure. The SPDF in (b) has more energy in the longer wavelengths ($\lambda > 550$ nm), corresponding to yellow and red, and appears orange to the eye. The actual colour is shown in the inset rectangle. The spectrum is that of Munsell colour 10YR 7/12.

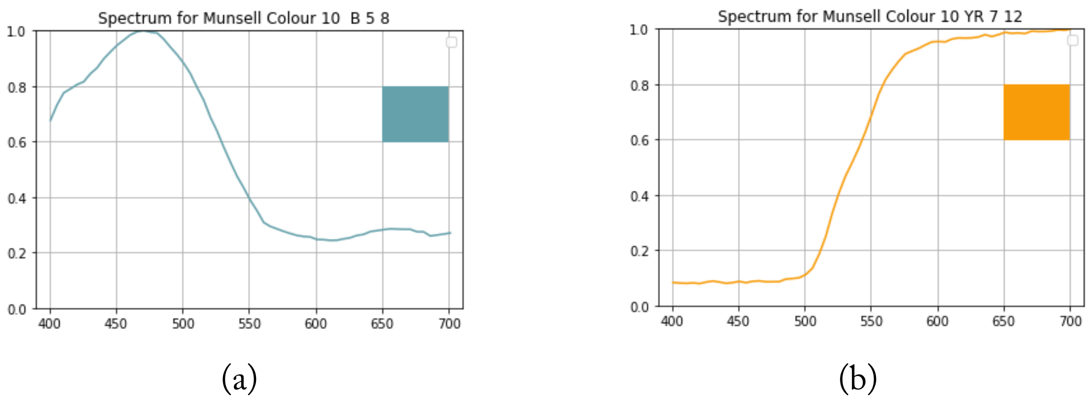

Figure 1.2: *Colour is the human-vision system's interpretation of spectral power distribution functions. (a) SPDF of a blue colour (Munsell 10B 5/8) and the actual colour in the rectangle. (b) SPDF of an orange colour (Munsell 10YR 7/12) and the actual colour in the rectangle. X-axis is wavelength (in nm) and Y-axis is normalised energy.*

Electromagnetic energy in the visible range or light interacts with matter in several ways to produce various SPDFs and colours. In his book, Kurt Nassau[**?**] talks of fifteen causes of colour. However, there are four important physical processes by which colour is ordinarily produced in nature. The first is *emission*. Sufficiently hot objects radiate energy in visible wavelengths. The best example is our Sun whose surface is approximately 5600 K and emits radiation with a peak around 560 nm. The second is *absorption*. Most objects selectively absorb (or *scatter*) light incident on them. For example, a leaf appears green because it absorbs red and blue wavelengths and scatters green from the sunlight incident on it. The third is *transmission* where a surface is transparent to certain wavelengths while it blocks others. The fourth is *dispersion* in which different wavelength components travel at different velocities (*phase* velocities to be precise) through a medium. Such differences in combination with other optical phenomena, typically *refraction*, lead to separation of the incident light into its component wavelengths (and, therefore, colours). Creating a rainbow or a spectrum[1] using a prism is a classic example of this phenomenon.

1.2 Colour by Emission

Any object with a temperature greater than *absolute zero* – 0 Kelvin or about $-273.15°$ C – emits electromagnetic radiation called *thermal radiation* due to the kinetic energy of its constituent atoms and molecules. The SPDF depends on the temperature and heat-transfer characteristics given by

$$\alpha + \rho + \tau = 1$$

where α, ρ and τ are the spectral absorption, reflection and transmission components respectively. An object is called a *black-body* if $\alpha = 1$, that is, a perfect absorber.

Also, $\alpha = \epsilon$ where ϵ is the spectral emissivity component. Any object which absorbs a certain wavelength will also emit the same wavelength if heated sufficiently. It also means that a black-body is a perfect emitter, i.e. $\epsilon = 1$, when heated since its absorption component $\alpha = 1$. The relationship between absorption and emission is illustrated in Figure 1.3 showing the emission and absorption spectra of sodium. Low pressure, hot sodium vapour emits light at the famous *doublet* of 589.0 and 589.6 nm. These two wavelengths give the sensation of yellow colour to the human vision system and sodium light appears yellow. As these two wavelengths are also close to the wavelength peak, 560 nm, of the Solar spectrum, low pressure sodium vapour lamps have been and are still popular as street lamps at night although LED lights with their higher efficiency are becoming popular today. Figure (b) shows the spectrum when sunlight containing all visible wavelengths is passed through cool sodium vapour. It is seen that there are two black lines indicating that the sodium vapour absorbed these wavelengths from the incident sunlight. These two black lines are precisely at 589.0 and 589.6 nm showing that emission and absorption characteristics are identical.

Thermal radiation exhibits the following four properties:

1. Thermal radiation is not monochromatic, i.e. a single wavelength, but contains several wavelengths (and naturally, frequencies).

[1] The words spectrum and SPDF are used interchangeably throughout the text.

2. The power at each wavelength is different and the exact distribution, i.e. the SPDF, is given by *Planck's Law of Black-body Radiation*. The energy radiated (u) per unit area per unit solid angle is given by

$$u(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k T}} - 1} \tag{1.1}$$

where h is the Planck's constant and $= 6.626 \times 10^{-34}\ \mathrm{m^2\,kg\,s^{-1}}$, c is the velocity of light in vacuum ($= 3 \times 10^8\ \mathrm{m\,s^{-1}}$), k is the Boltzmann constant ($= 1.3806 \times 10^{-23}$ J/K) and T is the temperature in K.

3. There is a peak wavelength at which maximum power is radiated or absorbed. This peak wavelength is given by Wien's Displacement law

$$\lambda_{\mathrm{max}} = \frac{b}{T} \tag{1.2}$$

where b is Wien's displacement constant $= 2.898 \times 10^{-3}$ m K.

4. The total amount of energy grows as the *fourth* power of temperature and is given by the Stefan-Boltzmann law

$$E = \sigma A\, T^4 \tag{1.3}$$

where σ is the Stefan-Boltzmann constant $= 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$.

These four properties are important in imaging self-luminous objects such as stars, light bulbs and fires.

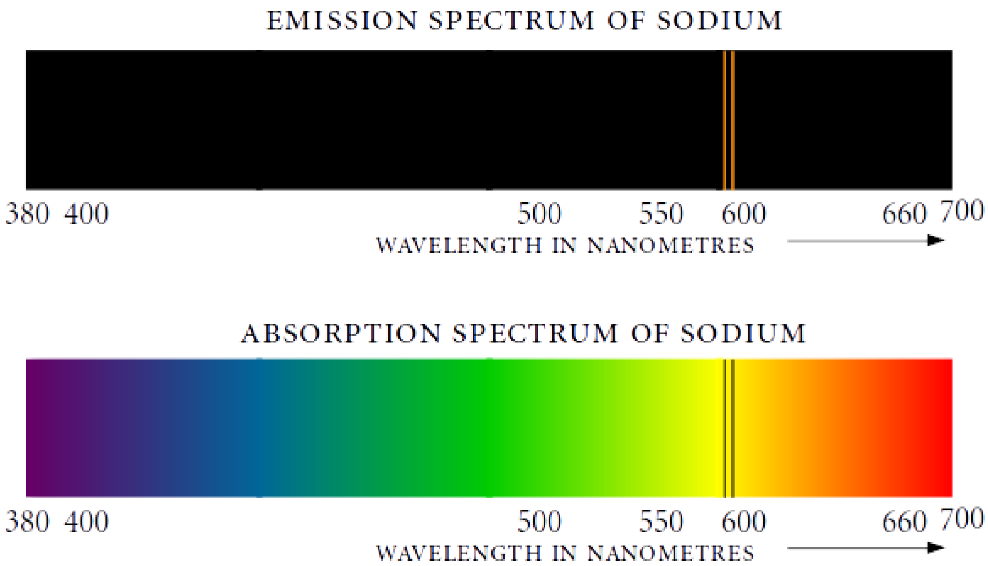

Figure 1.3: *Emission (top) and absorption (bottom) spectra of sodium vapour. Sodium emits at its characteristics wavelengths of* 589.0 *and* 589.6 *nm. When cold, it absorbs the same wavelengths.*

1.3 Colour by Absorption

Most opaque objects in nature get their colour by selectively absorbing certain wavelengths in the incident light and scattering the rest. Trees, fruits, vegetables, dust, birds, animals and almost everything else owe their colours to absorption processes. *Pigments* with specifically chosen absorption characteristics are added to man-made, synthetic substances and objects for producing desirable colours. Wall paints, lipsticks, fabric dyes, etc. are examples of coloured human-made objects.

An object P is irradiated by a light source having an SPDF given by $S_i(\lambda)$. P selectively absorbs and scatters the incident spectrum such that a new spectrum $S_r(\lambda)$ is produced. This is the radiated energy from P. $S_r(\lambda)$ is a complex function of $S_i(\lambda)$ depending on the nature of the constituent material, geometry and direction of viewing as well as incidence.

A simpler relationship that deals only with *total energy* is used extensively in Computer Graphics and Image Processing. The total energy of the source

$$E_i(\theta_i, \phi_i) = \int_0^\infty S_i(\lambda)\, d\lambda \tag{1.4}$$

is incident upon an object surface patch P in the direction (θ_i, ϕ_i). θ stands for the altitude and ϕ is azimuth characterising the specific direction in 3-D and are

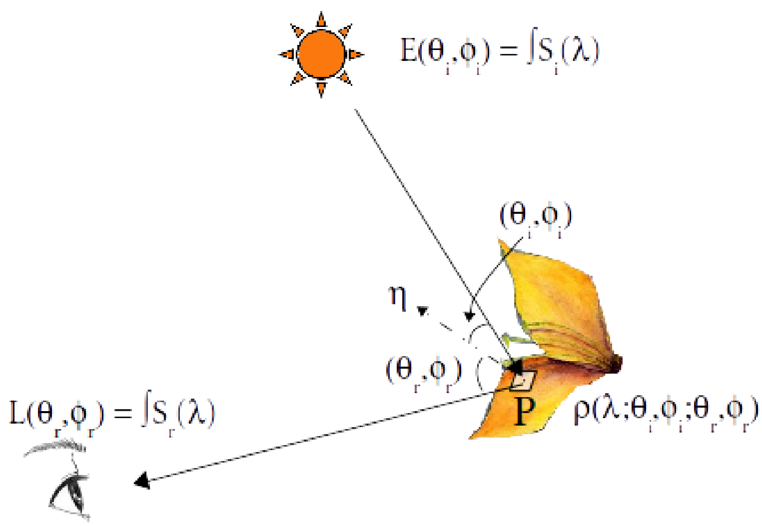

Figure 1.4: *Modelling colour by absorption: an object patch P is illuminated by a luminous source with SPDF $S_i(\lambda)$ from a direction (θ_i, ϕ_i) and viewed by an observer in direction (θ_r, ϕ_r).*

measured with respect to the surface normal η (Figure 1.4). The total energy

$$L(\theta_r, \phi_r) = \int_0^\infty S_r(\lambda)\, d\lambda \tag{1.5}$$

scattered in the direction (θ_r, ϕ_r) of an observer is then given by

$$L(\theta_r, \phi_r) = \rho\, B(\theta_i, \phi_i; \theta_r, \phi_r) E_i(\theta_i, \phi_i) \tag{1.6}$$

where ρ is the *albedo* or the reflection coefficient and B is known as the *Bidirectional Reflectance Distribution Function (BRDF)*. BRDF is so-called because it shows the effect of the two directions–incident and viewing–on the appearance of the object.

$$B(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L(\theta_r, \phi_r)}{E_i(\theta_i, \phi_i)} \tag{1.7}$$

A high albedo object scatters most of the energy incident upon it, e.g. snow ($\rho = 0.9$). A low albedo object appears dark and scatters very little of the incident light. Our Moon is a wonderful example having an albedo of only 0.07. It is one of the darkest knows bodies in the universe! Soot and lamp-black are more familiar examples of low albedo materials.

Two special limiting cases of BRDF are commonly used. When P appears the same in all directions, i.e., scatters light uniformly in all directions, it is called a *Lambertian Surface*. A Lambertian surface is a perfect diffuse reflector and the brightness appears same regardless of the direction of the observer. The BRDF for a Lambertian surface is a constant

$$B_L(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{1}{\pi} \tag{1.8}$$

The second case is when P behaves like a mirror. It scatters all the incident energy only in the one direction given by the law of reflection that states, "angle of incidence = angle of reflection". In this case,

$$B_L(\theta_i, \phi_i; \theta_r, \phi_r) = \begin{cases} 1 & \text{if } i = r \\ 0 & \text{otherwise} \end{cases} \tag{1.9}$$

BRDF captures the effect of geometry on appearance and is used in calculating how bright an object looks under different lighting and viewing conditions. However, it is not sufficient to capture colour appearance and variations under different lighting conditions. There are several generalisations of BRDF but note that none of them is standardised. The most general version is the *Bidirectional Scattering Distribution Function*(BSDF) (you can look it up on Wikipedia).

Equation 1.6 may be modified to handle colour variations. In the general case, we may add λ terms in the equation and get

$$L(\theta_r, \phi_r, \lambda_r) = \rho\, B(\theta_i, \phi_i, \lambda_i; \theta_r, \phi_r, \lambda_r) E_i(\theta_i, \phi_i, \lambda_i)$$

where λ_r and λ_i are the reflected/scattered and incident wavelengths respectively. However, most objects do not disperse and change the incident wavelength but only absorb some of the energy at that wavelength, i.e., we can safely replace λ_i and λ_r with λ. Thus, we get the following useful equation for colour scattering.

$$L(\theta_r, \phi_r, \lambda) = \rho\, B(\theta_i, \phi_i; \theta_r, \phi_r; \lambda) E_i(\theta_i, \phi_i, \lambda) \qquad (1.10)$$

Equation 1.10 may be further simplified for a Lambertian surface as

$$S_r(\lambda) = \frac{\rho}{\pi}(1 - P(\lambda))S_i(\lambda) \qquad (1.11)$$

where $S_r(\lambda)$ and $S_i(\lambda)$ are the incident and scattered SPDFs and $P(\lambda)$ captures the wavelength-dependent absorption properties of the scattering surface.

The use of Equation 1.11 is shown in Figure 1.5. The image of a leaf is shown in (a). SPDF of the incident sunlight (D_{65}) is in (b). The absorption characteristics $P(\lambda)$ of the leaf are shown in (c). A high value indicates that most of the energy is absorbed at that wavelength while a low value means that most of the energy is scattered. As is expected of a green leaf, $P(\lambda)$ has high values in the blue and red regions and low values between 500 and 580 nm. If we take $\rho/\pi = 1$, then the resulting spectrum after sunlight is scattered by the leaf is given in (d). The spectrum in (d) is obtained by using Equation 1.11 with $S_i(\lambda) = D_{65}$ and $P(\lambda)$ as the SPDF in (b). The colour, corresponding to the SPDF in (d) is also shown as the green-coloured rectangle and this colour corresponds to the colour of the area of the leaf in (a).

A more general and complex case is that of objects that *change their colour* depending on the viewing direction, e.g. certain dress materials and sarees, peacock feathers, colours caused by oil slicks, etc. Equation 1.10 has to be used if we want to model such objects. Luckily, in this book, we stay away from such objects for much of the time (although we do refer to them again in Chapter **??**!)

1.4 Colour by Transmission

Objects which allow light to pass through them are called transparent objects and the passage of light through such materials is *transmission*. When electromagnetic radiation of certain wavelengths strikes a transparent object, they do not resonate with the natural vibrations of the atoms and molecules in it. Instead these wavelengths cause layers of atoms to vibrate for a short time and pass them on through

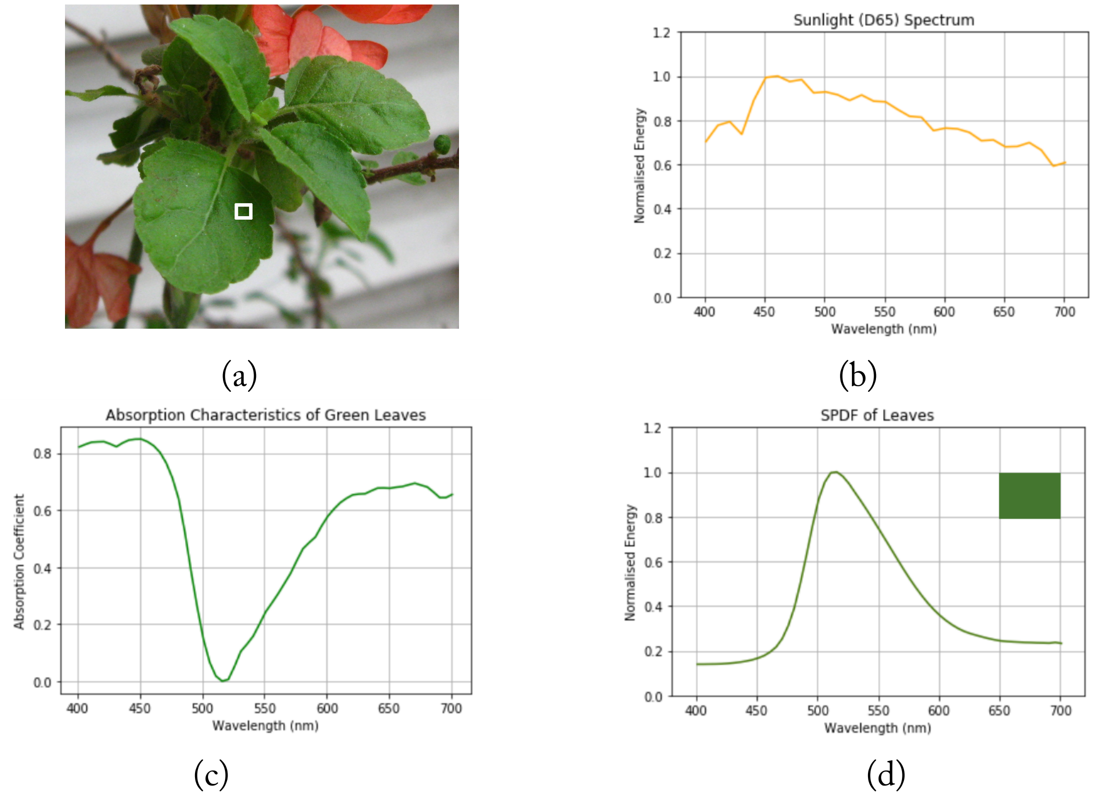

Figure 1.5: *Colour by absorption. (a) Leaves are green because they selectively absorb incident electromagnetic energy. (b) SPDF of standard sunlight (D_{65}). (c) Absorption characteristics of the region outlined in white in (a). (d) Resulting SPDF of D_{65} scattered by leaves shows a peak in the 500 - 580 nm wavelength region. Its corresponding colour is shown in the rectangle.*

the bulk of the material eventually re-emitting the incident wavelengths on the other side. Transparent objects appear clear or have a uniform colour.

Colour by transmission is a special case of colour by absorption. The role played by albedo is played by transmission coefficient τ. Most materials transmit different wavelengths differently and the transmission function may be modelled as $\tau(\lambda)$. Thus, the colour by transmission equation is analogous to the colour absorption equation (Equation 1.10) with $\rho(\lambda)$ replaced by $\tau(\lambda)$.

$$S_r(\lambda) = \tau(\lambda) S_i(\lambda) \tag{1.12}$$

In the special case of transmission, we assume that the angle of incidence is equal to the angle of viewing $\theta_i = \theta_r$ and the BRDF is '1'. Therefore, the colour resulting from transmission through a medium is a point-wise multiplication of the incident (source) spectrum $S_i(\lambda)$ with the transmission function $\tau(\lambda)$. Note also that, in most cases, we can take $\tau(\lambda) = 1 - P(\lambda)$.

Figure 1.6(right) shows the effect of viewing the image on the left through a blue filter. $\tau(\lambda)$ has high values at wavelengths corresponding to blue colour and

Figure 1.6: *Colour by transmission: original image is on the left. The same image, when seen through a blue-coloured glass, is shown on the right.*

low values at other wavelengths. As a result, the white areas in the original image appear bluish and the reds become purples.

1.5 Colour by Dispersion

Dispersion and refraction result in rainbows and mirages. Dispersion refers to the phenomenon where the velocity (or more precisely the *phase* velocity) of light or any other wave depends upon its frequency. The media where this phenomenon occurs are called *dispersive* media. Glass is a typical example of a dispersive medium.

The phase velocity, v is given by

$$v = \frac{c}{n} \tag{1.13}$$

where c is the velocity of light in vacuum and n is the refractive index. In dispersive media, $n = n(\lambda)$ and refractive index decreases with increasing wavelength.

When light travels from one material to another with a different refractive index, Snell's law of refraction states that light changes its direction at the boundary between such materials.

$$\frac{\sin\ \theta_1}{\sin\ \theta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2} \tag{1.14}$$

where θ_1 is the angle of incidence and θ_2 is the angle of refraction; n_1 and n_2 are the refraction indices of the two materials; and, v_1 and v_2 are the phase velocities of light in the two materials.

Figure 1.7 shows how dispersion produces a rainbow. Water droplets in the air cause sunlight to refract at their boundaries. If the angle of incidence is sufficiently high ($> 50°$, as is the case during early mornings and before sunset), the sunlight after refraction suffers total internal reflection within the water droplet and exits back towards the observer on the surface. The sunlight again undergoes refraction, this time at the droplet-air boundary, that further separates the constituent colours

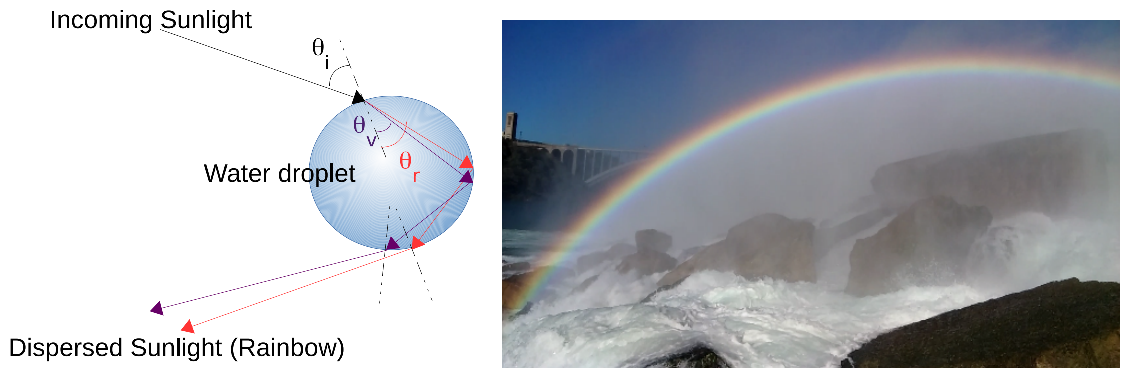

Figure 1.7: *Formation of a rainbow as an example of colour by dispersion*

and we get to see a rainbow. The red colour is always at the top because it deviates the least.

Another example of dispersion is *chromatic aberration* where the image of an object appears surrounded by coloured fringes. Chromatic aberration is common in long focal length convex lenses and leads to low-quality images in a telescope. Refracting telescopes (i.e., convex lens based ones) usually use a combination of lenses with opposing dispersion effects to minimise chromatic aberration.

1.6 Measurements on SPDF

Colour sensation is determined by the SPDF of incident light. There are five terms used in characterising an SPDF, namely, *intensity*, *brightness*, *hue*, *saturation* and *lightness*. The first four are directly related to and measured from the spectrum

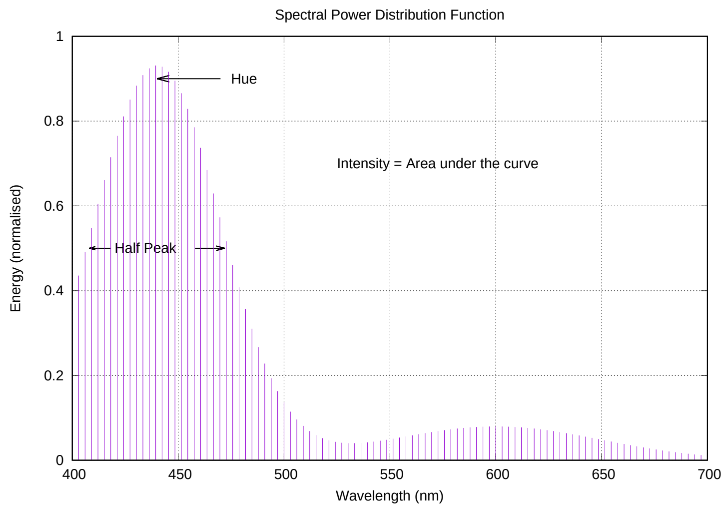

Figure 1.8: *Intensity, hue and saturation as may be measured on a spectrum*

whereas the last is entirely based on human perception.

Intensity: In Physics, intensity is the total amount of energy received by a surface per unit area. It is measured in Watts/m^2. It is more commonly called *irradiance* in non-optical communities. A closely related term is *luminous intensity* which is specific to the energy in visible wavelengths. Luminous energy is measured in *candela / steradian*. Remember that steradian is a measure of the 3D solid angle. We can measure it as the area under an SPDF.

Brightness: It is a *subjective* property of visual perception and refers to the appearance of an object that is radiating or scattering light. Brightness is dependent in a highly complex manner on the physical properties of the object. A loose way of thinking about it is that it is the result of the interaction between the incident light and the scattering properties of the object. It is also given by the area under the SPDF.

Hue: Hue refers to the colour of the object or the light. It is related to the dominant wavelength in the spectrum. It is found in a *unimodal* SPDF as the wavelength corresponding to the peak.

Saturation: Saturation is a measure of how concentrated the energy is around the peak. It is measured by the *half-peak width*, i.e., the width of the interval where the SPDF falls to half the value at the peak. Saturation is inversely proportional to the width: the wider the peak, the lower the saturation.

Figure 1.8 illustrates these terms. Intensity is given by the area under the curve and evaluates to approximately 80 units. Hue is 440 nm indicating a blue colour while the half-peak width is 64 nm. If we express it as a ratio of the total width of the SPDF which is 300 nm, then saturation may be calculated as 0.7867 (1 - 64/300).

1.7 Summary

This chapter discussed the physics behind colour and the physical processes responsible for producing colours. There are four important mechanisms by which colour is produced: emission by hot, luminous objects; absorption, by far the most dominant mode, and the reason for the colours we see around us; transmission, which results in the colours we see through windows, sun-glasses and other transparent media; and, dispersion which produces rainbows and certain undesirable effects such as chromatic aberrations. A proper understanding of these physical processes is important if one were to design and understand colour image processing operations that are meaningful and have a wide range of applications.

Exercises

1. A star radiates four times more energy than our Sun. If our Sunlight has a peak emission wavelength of 560 nm, what is the peak emission wavelength of the star? What colour does it appear?

2. Our Sun's surface temperature is 5800 K. Compute the SPDF of Sunlight assuming the Sun to be a black-body emitter.

3. Assume that an average human being can see wavelengths between 400 nm and 720 nm. What are the minimum and maximum temperatures required to produce colours that we can see?

4. Look up the WWW for *Draper Point*. What is the peak emission wavelength corresponding to Draper Point? How much energy does a black-body with a temperature equal to Draper Point emit in the visible range (assume visible range is 400 nm to 720 nm)?

5. D_{65} sunlight (Appendix **??**) is incident upon an object with a Lambertian surface from a direction given by ($\theta = 30°, \phi = 60°$). The object has an SPDF given by a Gaussian with ($\mu = 520\,\text{nm}, \sigma = 10\,\text{nm}$) when viewed from the direction ($\theta = 60°, \phi = 160°$). What is the actual SPDF of the object?