Department of Computer & Information Sciences
ALGORITHMICS (M.Tech AI) MINOR 1

22 August 2011 Total Marks: 20 Duration: 1 1/2 hours

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1. **[3 + 2 Marks]**

   (a) Order the following functions by their rate of growth. Partition the list into equivalence classes such that $f(n)$ snd $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.

   $$(3/2)^n,\ n^3,\ lg(n),\ 2^{logn},\ 2^n,\ nlogn,\ n,\ 2n$$

   (b) Prove that $2n + (3/2)logn = \Theta(n)$.

2. **[1+ 2+ 2 Marks]**

   (a) Explain optimality principle.

   (b) Give an example of a problem where optimality principle does not hold.

   (c) In the 0/1-knapsack problem suppose the items can be ordered such that profits(p) are in decreasing order with corresponding weights(w) in increasing order with p/w in decreasing order, can you propose a solution more efficient than the usual dynamic programming method? Explain.

3. **[6 Marks]** Propose three algorithms to solve the Select-Min problem. Analyze the efficiency of your algorithms.

4. **[4 Marks]**

   A. A permutation on the set of integers $A_N = \{1, 2, \ldots, N\}$ is an ordered sequence $\{a_1, a_2, \ldots a_N\}$ in which each integer a_i from the set A appears exactly once. For example, (3,1,2) is a permutation of A_3.

      i. Design an algorithm which, given an integer N and a sequence P of integers of length N, checks whether P is a permutation of A_N.

      ii. Analyze the time complexity for your algorithm.

   OR

   B. A *d-ary heap* is like a binary heap, but instead of 2 children, nodes have d children.

      i. How would you represent a d-ary heap in an array?

      ii. Derive the height of a d-ary heap of n elements in terms of n and d.

      iii. Describe an implementation for **Extract-Min**. Analyze its running time in terms of d and n.

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